

2.33. Suppose you are given $n \times n$ matrices A, B, C and you wish to check whether $AB = C$. You can do this in $O(n^{\log_2 7})$ steps using Strassen's algorithm. In this question we will explore a much faster, $O(n^2)$ randomized test.

- (a) Let \mathbf{v} be an n -dimensional vector whose entries are randomly and independently chosen to be 0 or 1 (each with probability $1/2$). Prove that if M is a non-zero $n \times n$ matrix, then $\Pr[M\mathbf{v} = \mathbf{0}] \leq 1/2$.
- (b) Show that $\Pr[AB\mathbf{v} = C\mathbf{v}] \leq 1/2$ if $AB \neq C$. Why does this give an $O(n^2)$ randomized test for checking whether $AB = C$?