Robots with Lights: Overcoming Obstructed Visibility Without Colliding

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Abstract. *Robots with lights* is a model of autonomous mobile computational entties operating in the plane in Look-Compute-Move cycles: each agent has an externally visible light which can assume colors from a fixed set; the lights are persistent (i.e., the color is not erased at the end of a cycle), but otherwise the agents are oblivious. The investigation of computability in this model, initially suggested by Peleg, is under way, and several results have been recently established. In these investigations, however, an agent is assumed to be capable to see through another agent.

In this paper we start the study of computing when visibility is obstructable, and investigate the most basic problem for this setting, *Complete Visibility*: The agents must reach within finite time a configuration where they can all see each other and terminate. We do not make any assumption on a-priori knowledge of the number of agents, on rigidity of movements nor on chirality. The local coordinate system of an agent may change at each activation. Also, by definition of lights, an agent can communicate and remember only a constant number of bits in each cycle. In spite of these weak conditions, we prove that COMPLETE VISIBILITY is always solvable, even in the *asynchronous* setting, without collisions and using a small constant number of colors. The proof is constructive. We also show how to extend our protocol for COMPLETE VISIBILITY so that, with the same number of colors, the agents solve the (non-uniform) CIRCLE FORMATION problem with obstructed visibility.

1 Introduction

1.1 Framework

In the traditional model of distributed computing by mobile entities in the plane, called *robots* or *agents*, each entity is modelled as a point; it is provided with a local coordinate system (not necessarily consistent with that of the other agents); it has sensorial capabilities, called *vision*, enabling it to determine the position (within its own coordinate system) of the other agents. The agents are anonymous, they are indistinguishable, and they execute the same code.

Agents operates in *Look-Compute-Move* cycles: when becoming active, an agent uses its sensing capabilities to get a snapshot of its surroundings (Look), then this snapshot is used to compute a destination point (Compute), and finally it moves towards

this destination (Move); after that, the agent becomes inactive. In the majority of investigations, the agents are assumed to be oblivious: at the beginning of each cycle, an agent has no recollection of its past observations and computations [11]. Depending on the assumptions on the activation schedule and the duration of the cycles, three main settings are identified. In the *fully-synchronous* setting, all agents are activated simultaneously, and each cycle is instantaneous. The *semi-synchronous* setting is like the fully synchronous one except that he set of agents to be activated is chosen by an adversary, subject only to a fairness restriction: each agent will be activated infinitely often. In the *asynchronous* setting, there is no common notion of time, and no assumption is made on timing of activation, other than fairness, nor on the duration of each computation and movement, other than it is finite.

Vision and mobility provide the agents with *stigmergy*, enabling the agents to communicate and coordinate their actions by moving and sensing their relative positions. The agents are otherwise assumed to be *silent*, without any means of explicit direct communication [11]. This restriction enables deployment in extremely harsh environments where communication is not possible, i.e an underwater deployment or a military scenario where wireless communication are impossible or can be jammed. Nevertheless, in many other situations it is possible to assume the availability of some sort of direct communication. The theoretical interest is obviously for weak communication capabilities.

A model employing a weak explicit communication mechanism is that of *robots with lights*: in this model, each agent is provided with a local externally visible *light*, which can can assume colors from a fixed set; the agents explicitly communicate with each other using these lights [5, 6, 10, 12, 14, 16]. In this model, the lights are persistent (i.e., the color is not erased at the end of a cycle), but otherwise the agents are oblivious.

The classical model of silent entities and the more recent model of entities with visible lights share a common assumption, that *visibility is unobstructed*. That is, three or more collinear agents are assumed to be mutually visible. It can be easily argued against such an assumption, and for the importance of investigating computability when visibility is obstructed by presence of the agents: given three collinear agents, the one in the middle blocks the visibility between the other two and they cannot see each other.

Nothing is known on computing with *obstructed visibility* except for the investigations on the so-called *fat agents* model, where agents are not points but unit discs, and collisions are allowed[5] and can be used as an explicit computational tool. (e.g., [1,2,4]); and for the study of uniformly spreading agents operating in a one dimensional space (i.e., on a line) [3]. In this paper we start to fill this void, and focus on agents with visible lights in presence of obstructed visibility.

The problem we investigate is perhaps the most basic in a situation of obstructed visibility, and it is the one of the agents reaching a configuration of complete unobstructeded visibility. More precisely, this problem, that we shall call COMPLETE VISIBILITY, requires the agents, starting from an arbitrary initial configuration where

[5] In pointilinear models, collisions create unbreakable symmetries; thus, unless this is the required outcome of the problem, their avoidance is required by all solution protocols. In addition, in real world implementations, collisions (e.g., of two quad copters) may have unpredictable outcomes that might be better avoided.

they are in distinct points but might be unable to see everybody and might not know the total number of agents[6], to reach within finite time a configuration in which every agent is in a distinct location from which it can see all other agents, and no longer move.

Among the configurations that achieve complete visibility, a special class is that where all agents are on the perimeter of a circle (not necessarily equally spaced). The problem of forming any such a configuration is called CIRCLE FORMATION and it has been extensively studied both in the classical model of silent agents and in the ones with visible lights (e.g., [7–9, 13, 15]). Unfortunately, none of these investigations consider obstructed visibility, and their algorithms do not work in the setting considered here.

1.2 Our Contributions

In this paper we study solving COMPLETE VISIBILITY by robots with lights. That is, we consider autonomous and anonymous agents, each endowed with a visible light that can assume a constant number of persistent colors, that are otherwise oblivious, and whose visibility is obstructed by other agents in the line of sight; and we investigate under what conditions they can solve COMPLETE VISIBILITY and at what cost (i.e., how many colors).

We do not make any assumptions on a-priori knowledge on the number of agents, nor on agreement on coordinate systems, unit of distance and chirality; actually, the local coordinate system of an agent may change at each activation. Neither we make any assumption on rigidity of movements; that is, a move may be stopped by an adversary before the agent reaches its destination; the only constraint is that, if interrupted before reaching its destination, the agent moves at least a minimum distance $\delta > 0$ (otherwise, no destination can ever be reached). Also, by definition of lights, an agent can communicate and remember only a constant number of bits in each cycle.

In spite of these weak conditions, we prove that COMPLETE VISIBILITY is always solvable, even in the *asynchronous* setting, without collisions and using a small constant number of colors. The proof is constructive. We first design a protocol that achieves complete visibility with six colors under a semi-synchronous scheduler. We then show how to transform it into an asynchronous algorithm with only four additional colors. We also show how to extend the protocol so that, under the same weak conditions and without increasing the number of colors, the agents can position themselves on the perimeter of a circle. In other words, we also show how to solve the (non-uniform) CIRCLE FORMATION problem with obstructed visibility.

Due to lack of space, some of the proofs are sketched and some omitted.

2 Model and Definitions

Consider a set of mobile anonymous agents $\mathcal{A} : \{a_1, a_2, .., a_n\}$. Each agent a_i has a persistent state variable s_i, which may assume any value in a finite set of *colors* C.

[6] The actual number of agents may be unknown for several reasons; e.g., if the deployment of agents has been done by an airplane, a subset of agents may be lost or destroyed during the landing process.

We denote by $x_i(t) \in \mathbb{R}^2$ the position occupied by agent a_i at time t expressed in some global coordinate system (used only for description purposes, and unknown to the agents); when no ambiguity arises, we omit the indication of time. A *configuration* $\mathcal{C}$ is a set of n tuples in $C \times \mathbb{R}^2$ each defining the position and color of an agent; let $\mathcal{C}_t$ denote the configuration at time t.

Each agent a_i has its own system of coordinates centered in itself, which does not necessarily agree with those of the other agents, i.e. there is no common unit of measure and not common notion of clockwise orientation. Agents a_i and a_j are visible to each other at time t if and only if the segment $\overline{x_i(t)x_j(t)}$ does not contain any other agents. Let $\mathcal{C}_t[a_i]$ denote the set of the positions and colors of the agents visible to a_i time t. We shall call such a set *local view*. A configuration $\mathcal{C}$ is said to be *obstruction-free* if $\forall a_i \in \mathcal{A}$ we have $|\mathcal{C}[a_i]| = n$; that is, if all agents can see each other. Two agents a_i and a_j are said to *collide* at time t if $x_i(t) = x_j(t)$.

At any time, agents can be active or inactive. When activated, an agent a_i performs a sequence of operations called *Look-Compute-Move*: it activates the sensors to obtain a snapshot (called *local view*) of the positions of the visible agents expressed in its own coordinate system (*Look*); it then executes an algorithm (the same for all agents) based on its local view, which returns a destination point $x \in \mathbb{R}^2$ and a color $c \in C$ (*Compute*); it then sets its own state variable to c and moves towards x (*Move*). The movement may be stopped by an adversary before the agent reaches its destination; the only constraint on the adversary is that, if interrupted before reaching its destination, a robot moves at least a minimum distance $\delta > 0$ (otherwise, no destination can ever be reached).

We consider two schedulers for the activation of the agents: *Semi-Synchronous* (*SSYNC*) and *Asynchronous* (*ASYNC*). In *SSYNC*, the time is discrete; at each time instant t (called *round*) a subset of the agents is activated and performs its operational cycle instantaneously. The choice of the activation is done by an adversary, which however activates each agent infinitely often. In *ASYNC*, there is no common notion of time; each agent is activated independently, and each Compute and Move operation can take an unpredictable (but bounded) amount of time, unknown to the agent.

At the beginning (time t=0), the agents start in an arbitrary configuration $\mathcal{C}_0$ occupying different positions, and they are *black* (the state variable of each one is set to a special symbol $\mathfrak{b}$). The goal is for the agents to reach, in finite time, an obstruction-free configuration without ever colliding. We call this problem COMPLETE VISIBILITY. An algorithm is said to solve the problem if it always achieves complete visibility regardless of the choices of the adversary, and from any initial configuration.

Let $\mathcal{H}_t$ be the convex hull defined by $\mathcal{C}_t$, let $\partial\mathcal{H}_t = \mathcal{V}_t \cup \mathcal{B}_t$ denote the agents on the border of $\mathcal{H}_t$, where $\mathcal{V}_t : \{v_1, \ldots, v_k\} \subseteq \mathcal{A}$ is the set of agents (*corner-agents*) located at the corners of $\mathcal{H}_t$ and $\mathcal{B}_r : \{b_1, \ldots, b_l\}$ is the set of those located on the edges of $\mathcal{H}_r$ (*edge-agents*); let $\mathcal{I}_t$ be the set of agents that are interior of $\mathcal{H}_t$ (*interior-agents*). Let $n_t = |\mathcal{V}_t|$ be the number of corners in $\mathcal{H}_0$. Given an agent $a_i \in A$, we denote by $\mathcal{H}_t[a_i]$ the convex hull of its local view $\mathcal{C}_t[a_i]$. Let $\mathcal{C}_t^c$ indicate the set of agents in $\mathcal{C}_t$ with color c at time t, similarly we define $\mathcal{H}_t^c[a_i]$ as the convex hull, of $\mathcal{C}_t^c[a_i]$. Analogously defined are the extensions of $\mathcal{V}_t, \mathcal{B}_t, \mathcal{I}_t$. Given a configuration $\mathcal{C}$, we indicate by $SEC(\mathcal{C})$ the smallest enclosing circle containing $\mathcal{C}$ (when no ambiguity

arises we just use the term SEC). Given two points $x, y \in R^2$ with xy we indicate the line that contains them, and we use the operator $\cap$ to indicate the intersection of lines and segments. Let $d(x, y)$ indicate the Euclidean distance between two points (or a segment and a point); moreover, given $x, y, z \in R^2$ we use $\angle xyz$ to indicate the angle with vertex y and sides xy, yz. In the following, with an abuse of notation, when no ambiguity arises, we use a_i to denote both the agent and its position.

3 Complete Visibility in SSYNC

In this Section we provide an Algorithm that reaches Complete Visibility in the semi-synchronous setting. The algorithm is described assuming $|\mathcal{V}_0| \geq 3$; we will then show how the agents can easily move to reach this condition starting from a configuration with $|\mathcal{V}_0| = 2$.

Our algorithm works in two phases: (1) *Interior Depletion* (ID) and (2) *Edge Depletion* (ED). The purpose of the Interior Depletion phase is to reach a configuration $\mathcal{C}_{ID}$ in which there are no interior-agents. In this phase, the interior-agents move towards an edge they perceive as belonging to the border of the convex hull, and they position themselves between two corner-agents. At the end of this phase, all agents are on $\partial\mathcal{H}_0$. The goal of the Edge Depletion phase is to have all agents in $\mathcal{B}_{ID}$ to move so to reach complete visibility.

3.1 Phase 1: Interior Depletion Phase

Initially all agents are *black*. The objective of this phase is to have all agents on $\partial\mathcal{H}_0$, with the corner-agents colored *red* and the edge-agents colored *brown*.

Notice that corner (resp. edge) agents are able to recognize their condition in spite of possible obstructions. In fact, if a *black* agent a_i is activated at some round r, and it sees that $\mathcal{C}_r[a_i]$ contains a region of plane that is free of agents and wider than $180°$, then a_i knows it is a corner and sets its variable s_i to *red*. A similar rule is applied to edge-agents; in this case, an edge-agent a_i sets its variable s_i to *brown* if $C_r[a_i]$ contains a region of plane free of agents and wide exactly $180°$ (see Coloring Case of Figure 1).

In the ID phase, corner-agents color themselves *red*, and no longer move, while edge-agents color themselves *brown*. Each interior-agent a moves to position itself on one of its nearest visible edges of $\partial\mathcal{H}_0$; note that an edge of $\partial\mathcal{H}_0$ can be recognized in a's local view once it is occupied only by *brown* and *red* agents. To prevent collisions, the interior-agent moves towards the chosen edge e perpendicularly if and only if it is one with minimum distance to e and its destination on e is empty; otherwise it does not move. An edge-agent on the destination of an interior one, slightly moves to make room for the interior-agent. The INTERIOR DEPLETION algorithm is detailed in Figure 1.

It is easy to see that at the end of this phase, all the agents will be positioned on a convex hull.

Lemma 1. *For any initial configuration $\mathcal{C}_0$ there exists a round $r \in \mathbb{N}^+$ such that in $\mathcal{C}_r$ we have that $\mathcal{I}_r = \emptyset$; furthermore, this occurs without collisions.*

Algorithm INTERIOR DEPLETION (for the generic agent a_i activated at round r)

- Coloring Case: if ($s_i = black$) then:
  - If (a_i is a corner-agent in $\mathcal{H}_r[a_i]$)) then a_i sets $s_i = red$
  - If (a_i is an edge-agent in $\mathcal{H}_r[a_i]$)) then a_i sets $s_i = brown$
- Interior Case: if (a_i is interior in $\mathcal{H}_r[a_i]$) and $s_i = black$) then:
  - a_i uses its local view $\mathcal{C}_r[a_i]$ to determine the edges of $\partial\mathcal{H}_r[a_i]$.
  - If ($\exists e \in \partial\mathcal{H}_r[a_i]$ such that $\forall a_j \in \mathcal{I}_r[a_i], d(a_j, e) \leq d(a_i, e)$) then
    - a_i computes a point x of e such that $\overline{a_i x} \perp e$; if x is empty, then a_i moves toward x
- Obstructing Edge Case: if ($s_i = brown$) then:
  - Let e be the edge to which a_i belongs; if ($\exists a_j \in \mathcal{I}_r[a_i] \wedge \overline{a_j a_i} \perp e$), then a_i moves toward the nearest point $x \in e$ such that $\forall a_k \in \mathcal{I}_r[a_i], \overline{a_k x} \not\perp e$.

Fig. 1: Algorithm for the Interior Depletion Phase

Theorem 1. *There is a round $r \in \mathbb{N}^+$ such that the agents occupy different positions on $\mathcal{H}_r$. Moreover, the corner-agents are* red*, and the edge-agents are* brown*.*

3.2 Phase 2: Edge Depletion -ED

The purpose of the ED phase is to move the edge-agents out of the current convex hull to reach a final configuration whose convex hull includes $\mathcal{H}_0$ and all agents are on the corners, thus achieving complete visibility.

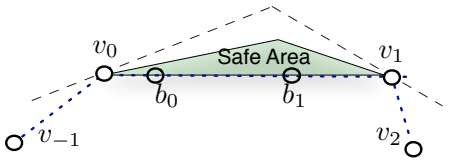
The algorithm makes an edge-agent move from its edge $e = \overline{v_0 v_1}$ to a point out of the current convex hull, but within a *safe zone*. Safe zones are calculated so to guarantee that *red* agents never cease to be located on corners of the current convex hull, in spite of the movement of the edge-agents. More precisely, the safe zone $S(e)$ of e consists of the portion of plane outside the current convex hull, such that $\forall x \in S(e)$ we have $\angle x v_0 v_1 < \frac{180^\circ - \angle v_{-1} v_0 v_1}{4}$ and $\angle v_0 v_1 x < \frac{180^\circ - \angle v_0 v_1 v_2}{4}$ (see Figure 2a).

Note that, due to the mutual obstructions that lead to different local views, edge-agents cannot always compute $S(e)$ exactly (see Figure 2b). In fact, only when there is a single edge-agent between the two *red* corner-agents on e, the computation of $S(e)$ is exact; in any case, we can show that the safe area $S'(e)$ computed by an agent is $S'(e) \subseteq S(e)$ and thus still safe.

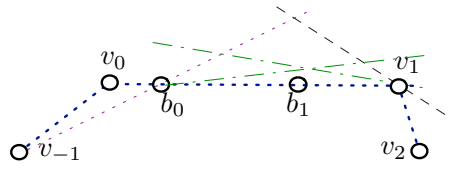
The migration of edge-agents and their transformation into corner-agents occurs in steps: in fact, if the edge e contains more than one edge-agent, our algorithm makes them move in turns, starting from the two agents b_1 and b_0 that are immediate neighbors of the corners v_1 and v_0, respectively. Only once they are out of the convex hull and they are corner of a new edge e', other agents on e will follow, always moving perpendicularly to e'. Careful changes of colors are required to coordinate this process. In fact, once the first pair is in position, the two agents will become *blue* to signal the other *brown* agents on e that it is their turn to move out; they will set their color to *red*

only when there is no interior-agent in the space delimited by e' and e. Once *red*, their color will never change until completion.

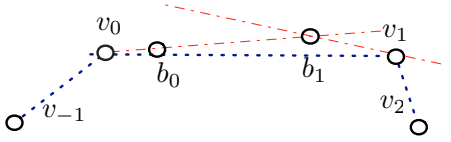
Due to the different estimations of S, to semi-synchronicity, and to the unpredictable distance traversed by an agent (possibly stopped before destination), a variety of situations could disrupt this ideal behaviour. In particular, it could happen that only one of the two agents, say b_1, moves while the other stays still, or that b_1 moves further from e than b_0. In both cases this leads to a configuration in which b_0 becomes a interior or edge-agent. This problem is however adjusted by b_1 that, when noticing the situation, moves towards v_1 until b_0 becomes a corner in $\mathcal{H}[b_1]$. A further complication is that b_1 might wrongly perceive b_0 as a corner and thus decide not to move; this occurs if v_0b_0 happens to be collinear with b_1 obstructing visibility; such a case is however detected by b_0 itself, which uses a different color (*orange*) to signal that b_1 has to move further towards v_1 to transform b_0 into a corner (see Figure 2d).



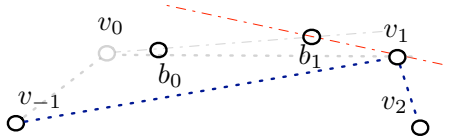
(a) Safe Area of edge $\overline{v_0v_1}$: an agent moving inside the safe area cannot create collinearity with agents on the neighboring edges



(b) Approximation of the safe area computed by agent b_1 using as reference the two lines v_1v_2 and $v_{-1}b_0$. This approximation is entirely contained in the real safe area



(c) Creation of a new edge, due to non-rigid movements or to different approximations of the safe area, agent b_1 could move making agent b_0 interior, this condition is adjusted by letting b_1 move towards v_1



(d) b_1 could move in such a way to become collinear to v_0b_0, b_0 signals this condition by changing its color

Fig. 2: Edge Depletion Phase

The detailed algorithm for the ED phase is reported in Figure 3.

3.3 The case of $|\mathcal{V}_0| = 2$

The strategy of the previous Section works for $|\mathcal{V}_0| > 2$. It is however simple to have the agent move to reach such a condition from $|\mathcal{V}_0| = 2$, as described below.

When $|\mathcal{V}_0| = 2$ the agents are necessarily disposed forming a line and $|\mathcal{A}| \geq 2$. First notice that an agent a can detect that the configuration is a line, and whether it is an extremity (i.e., it sees only one other agents a'), or an internal agent (i.e., it is

Algorithm EDGE DEPLETION
For agent a_i activated at round r; to be executed if and only if $\nexists(black, a_j) \in \mathcal{C}_r[a_i]$.

- Execute COMPUTE ORDER and appropriate case from the list below.

- BROWN EDGE CASE: a_i belongs to an edge e of $\mathcal{C}_r[a_i]$ and $s_i = brown$.
  If a_i is the only agent on e then
  - a_i computes the angles $\alpha = 180^\circ - \angle v_{-1}v_0a_i$, $\beta = 180^\circ - \angle a_iv_1v_2$, and $\gamma = min(\frac{\alpha}{4}, \frac{\beta}{4})$; it then computes a point x such that $\angle xv_1a_i < \gamma$ and $\angle xv_0a_i < \gamma$.
  - a_i sets $s_i = yellow$.
  - a_i moves perpendicularly to e with destination x.

  If a_i is not the only non-*red* agent on e and one of its neighbors on e is *red* (by routine COMPUTEORDER, this agent is v_1) then: let b be its other neighbor;
  - a_i computes the two angles $\alpha = 180^\circ - \angle a_iv_1v_2$, $\beta = 180^\circ - \angle v_{-1}ba_i$, and $\gamma = min(\frac{\alpha}{4}, \frac{\beta}{4})$; it then computes a point x such that $\angle xv_1a_i < \gamma \wedge \angle xba_i < \gamma$.
  - a_i sets $s_i = yellow$.
  - a_i moves perpendicularly to e with destination x.
- YELLOW CASE: $s_i = yellow$.
  - if there is another *yellow* or *blue* agent a_j with $e^{a_i} = e^{a_j}$ then
    - if $a_iv_1 \cap a_jv_0 \notin (\overline{a_iv_1} \cup \overline{a_jv_0})$ then a_i sets $s_i = blue$
    - if $a_iv_1 \cap a_jv_0 \in \overline{a_iv_1}$ then a_i moves towards v_1 along $\overline{a_iv_1}$ of $\frac{d(a_i,v_1)}{2}$
    - if $a_j \in a_iv_1$ then a_i sets $s_i = orange$
  - else if $\nexists(s_j, a_j) \in \mathcal{C}_r[a_i]$ with $a_j \neq a_i$ and $e^{a_i} = e^{a_j}$ and $\nexists(s_j, a_j) \in \mathcal{C}_r[a_i] \cap e^{a_i}$ then
    - a_i set $s_i = red$
- ORANGE CASE: $s_i = orange$.
  - if there is another blue agent a_j with $e^{a_i} = e^{a_j}$ then
    - if $a_j \notin a_iv_1$ then a_i sets $s_i = blue$
- BLUE CASE: $s_i = blue$ and $a_i \in e$ with e edge of $\mathcal{C}_r[a_i]$.
  - if there is another orange agent a_j with $e^{a_i} = e^{a_j}$ then
    - a_i moves along $\overline{a_iv_1}$ in direction of v_1 towards the point at distance $\frac{d(a_i,v_1)}{2}$
  - else if $\nexists(brown, a_j) \in \mathcal{C}_r[a_i]$ such that a_j could move to e then
    - a_i sets $s_i = red$
- BROWN INTERIOR CASE: a_i is such that $s_i = brown$ and $a_i \in \mathcal{I}_r[a_i]$.
  - if there exists and edge $e' = \overline{a_xa_y}$ with $s_x = s_y = blue$ and a_i could move perpendicularly towards e' without crossing any segment delimited by two *red* agents, then a_i moves towards e'.
  - if $a_i \in e = \overline{a_0a_1}$ with $e \in \partial\mathcal{H}_r^{\{red,brown\}}[a_i]$ and $\exists x \in \mathbb{R}^2$ such that $a_0a_i \perp \overline{xa_i}$ and $\nexists a_j \in \angle a_0a_ix$ or $\nexists a_j \in \angle a_1a_ix$ then a_i executes the second subcase of the BROWN EDGE CASE.
- CORNER CASE: a_i is a corner of $\mathcal{C}_r[a_i]$ and $s_i = red$.
  - a_i can check local termination and the global termination
    - a_i locally terminates when $s_i = red$
    - a_i detects the global termination of ED phase when $\nexists(s_j, a_j) \in \mathcal{C}_r[a_i]$ with $s_j \neq red$

Fig. 3: Edge Depletion Phase algorithm

Procedure COMPUTE ORDER

- if a_i belongs to an edge e of $\mathcal{H}_r[a_i]$ and $s_i = brown$, it orders the *red* agents in its local view in a circular order, starting from the closest, $(v_1, v_2, \ldots, v_0)$.
- if $s_i \in \{orange, blue, yellow\}$, then a_i determines which of its current neighbors was v_1 in its previous computation and the edge $e^{a_i} = \overline{v_1 v_0}$ to which it belonged:
  - a_i computes the nearest edge $e = \{u, v\} \in \mathcal{H}_r^{red}[a_i]$
  - a_i computes the point $x \in \mathbb{R}^2$ such that is $uv \perp a_i x$
  - a_i sets $v_1 = u, v_0 = v$ if $\nexists a_j \in \angle u x a_i$ otherwise it sets $v_1 = v, v_0 = u$.
  - a_i sets $e^{a_i} = \overline{v_1 v_0}$

Color	**Meaning**	**Transition to:**
$Black$	initial color of all agents	$\{Red, Brown\}$
$Brown$	agents on edges or having to move to a new edge of $\mathcal{H}$	$Yellow$
$Yellow$	agents moving out of $\mathcal{H}$ to form a new edge	$\{Blue, Orange, Red\}$
$Orange$	agents needing to be transformed into corners	$Blue$
$Blue$	corner-agent now forming a new edge e, waiting for other agents to move to e	Red
Red	a stable corner-agent	$-$

Fig. 4: Colors used in the COMPLETE VISIBILITY algorithm.

between two collinear agents). If a is internal, it does not move; if it is an extreme, a sets its color to *red* and moves perpendicular to the segment $\overline{a'a}$. This means that, as soon as at least one of the extremes is activated, it will move (or they will move) creating a configuration with $|\mathcal{V}| > 2$. At that point, the algorithm previously described is applied.

Correctness of the ED phase. With the following lemma we show that the global absence of interior-agents with respect to the initial convex hull, can be locally detected by each agent.

Lemma 2. *Given an agent $a_i \in \mathcal{A}$ with $s_i \in \{red, brown\}$ and a round $r \in \mathbb{N}^+$, if $\nexists (black, a_j) \in \mathcal{C}_r[a_i]$ then $\mathcal{C}_r$ does not contain interior-agents with respect to $\mathcal{H}_0$.*

Proof. By contradiction, assume that $\nexists (black, a_j) \in \mathcal{C}_r[a_i]$ but there exists at least an interior-agent a with respect to $\mathcal{H}_0$. By the rules of the ID phase, agent a cannot change its color from $black$ to another because it can detect it is neither a corner nor a border. Thus, a is not in $\mathcal{C}_r[a_i]$ because $\mathcal{C}_r[a_i]$, by assumption, does not contain *black* agents. Thus, it must exist an agent a_k that has color different from $black$ and $a_k \in \overline{a_i a}$. But since a is interior then also a_k is interior, and so $s_k = black$. $\square_{Lemma\ 2}$

We now show that the safe area $S'(e)$ computed by an edge-agent on e is such that $S'(e) \subseteq S(e)$ and thus its movement is still safe (it does not transform a *red* corner into an interior or edge-agent).

Lemma 3. *Given a configuration $\mathcal{C}_r$ and an edge $e = \overline{v_0 v_1}$ of $\mathcal{H}_r$, if an agent $a_j \in e$ moves from e, it moves inside the safe zone $S(e)$*

Proof. The case when there is a single edge-agent $b \in e$ is trivial because b can compute exactly $S(e)$. Consider now the case when there are two or more edge-agents on e; among those, let b_0 and b_1 be the two that are neighbors of v_0, v_1. Those agents move only when executing the Brown Edge Case or Brown Interior Case. Let us consider the movement of the first that is activated, say b_1. Agent b_1 has two neighbors on e: a *brown* neighbor b and the *red* corner v_1. Agent b_1 orders the corners in its view from v_1 to v_{last}, according to its local notion of clockwise, where v_{last} is the last corner before b, i.e. v_{-1} in Figure 2b. Following the rules of the algorithm, b_1 computes: $\alpha = 180^\circ - \angle v_{last} b b_1$, $\beta = 180^\circ - \angle b_1 v_1 v_2$, and $\gamma = min(\frac{\alpha}{4}, \frac{\beta}{4})$. Angle $\angle v_{last} b b_1$ is an upper bound on $\angle v_{last} v_0 b_1$, otherwise we could get a contradiction since $v_{last} b$ and $v_{last} v_0$ will intersect in two points: one is v_{last} and the other one is after the intersection of $v_{last} v_0$ and $v_0 v_1$, that is impossible. Thus, α is a lower bound on the angle that a single agent would compute on e, which implies that b_1 will move inside $S(e)$. The same holds for b_0. Notice that, given two points x and y inside the safe zone, any point $z \in \overline{xy}$ is still inside the safe zone, thus any agent that moves on the lines connecting two agents inside $S(e)$ will still be in $S(e)$, completing the proof. $\square_{Lemma\ 3}$

The next lemma shows that the moves of our algorithm cannot transform any *red* corner-agents into an interior-agent.

Lemma 4. *Consider a corner-agent v_1 of $\mathcal{H}_{r'}$ with $s_1 = red$, we have that $\forall r \in \mathbb{N}^+$ with $r > r'$, v_1 is also a* red *corner-agent of $\mathcal{H}_r$.*

Proof. It is easy to see that during the ID phase we have that $\mathcal{H}_r = \mathcal{H}_0$ since the interior-agents will never trespass the edges of $\mathcal{H}_0$, so the hypothesis holds. We have to show that the same holds during the ED phase. We have that v_1 never moves after it sets $s_1 = red$ so if v_1 is a corner it cannot become interior as a consequence of its own move. Consider the two edges adjacent to v_1: $e_1 = \overline{v_0 v_1}$ and $e_2 = \overline{v_1 v_2}$. Assume, by contradiction, that there exists a round r in which the moves of a set X of agents on these two edges is such that v_1 is a corner-agent in $\mathcal{H}_{r-1}$ but not in $\mathcal{H}_r$. From Lemma 3 we have that agents in X move to points inside the safe zones $S(e_1)$ and $S(e_2)$ of e_1, e_2. Let us consider two points $x \in S(e_1)$ and $y \in S(e_2)$, such that agents on them will make v_1 interior. If v_1 is interior in $\mathcal{H}_r$, we have that $\angle x v_1 y > 180^\circ$. It is easy to see that $\angle v_0 v_1 x < \gamma$ (see Brown Edge Case and Brown Interior Case of Figure 3) and that $\gamma \leq \frac{180^\circ - \angle v_0 v_1 v_2}{4}$, since $\gamma = min(\frac{\alpha}{4}, \frac{\beta}{4})$, and that at least one of the two among β, α is a lower bound on $180^\circ - \angle v_0 v_1 v_2$. The same holds for y, so we have $\angle v_2 v_1 y \leq \frac{180^\circ - \angle v_0 v_1 v_2}{4}$. Thus, we have $\angle v_0 v_1 x + \angle v_2 v_1 y + \angle v_0 v_1 v_2 < 180^\circ$ and then $\angle x v_1 y < 180^\circ$, which is a contradiction. So, v_1 cannot be interior in $\mathcal{H}_r$. The same arguments hold if at round $r-1$ we consider a set of agents X on two edges e', e'' that are not adjacent to v_1; this is easy to see since, given $x \in S(e')$ and $y \in S(e'')$ we

have $\angle x v_1 y \leq \angle v_0 v_1 v_2 < 180°$, which is another contradiction to the hypothesis of v_1 being interior in $\mathcal{H}_r$. $\square_{Lemma\ 4}$

In the next sequence of lemmas, we show that, given an edge e in a configuration $\mathcal{C}$ of the ED phase, all edge-agents in e will eventually became *red* corners.

Lemma 5. *Given a configuration $\mathcal{C}_r$ and an edge e of $\mathcal{H}_r$ with a single brown agent b on e, eventually b will be a* red *corner.*

Proof. Since *red* corners never move and no interior-agents can be moving on e, while inactive, agent b maintains its single position inside e. When activated at some round r', agent b executes the Brown Edge Case with a single agent. Thus b switches color to *yellow* and it moves perpendicularly to e of at least $min(d(v_h, x), \delta)$. At round $r'+1$, b is a corner-agent of $\mathcal{H}_{r'+1}$; in the next activation, after executing the Yellow Case code, b becomes *red*. $\square_{Lemma\ 5}$

Lemma 6. *Given a configuration $\mathcal{C}_r$ and an edge e of $\mathcal{H}_r$ with exactly two brown agents b_0, b_1 on e, eventually they will set their state variable to $yellow$ and they will move outside e.*

Proof. Let b_1 be the first to be activated at some round $r' \geq r$. At that time, b_1 switches its color to *yellow* and it moves perpendicularly to e (see Brown Edge Case). Agent b_0 will do the same, no matter if it is activated in round r' or in some successive rounds (see Brown Edge Case and Brown Interior Case). $\square_{Lemma\ 6}$

Lemma 7. *Given a configuration $\mathcal{C}$, any agent b_1 with $s_1 = yellow$ eventually becomes corner and will sets its state variable to red.*

Proof. If b_1 is *yellow* then a_1 has moved from an edge $e = \overline{v_0 v_1}$. If b_1 was not the only agent on e that could move, then there is (or there will be) another *yellow* agent b_0 moving from e. By construction, b_1 waits until it sees the other *yellow* agent b_0 (see Yellow Case). If both b_1 and b_0 realize to be corners of the current convex hull, then they eventually set their color to *blue* and then to *red*, thus the lemma is proved. However, due to the non-rigidity or the different local views of b_1 and b_0, the pathological case of Figure 2c may arise where one of the two, say b_0, becomes an interior-agent. This case is adjusted by the Yellow Case rule: each time a_1 is activated, it will move towards v_1 until a round r'' is reached when b_0 is not interior anymore in $\mathcal{C}_{r''}[b_1]$. Note that, since b_1 moves always half of the distance $d(b_1, v_1)$, and the number of rounds until the next activation of b_0 is finite, we have that b_1 will never touch v_1. Two possible sub-cases may happen at round r'': (i) $b_1 v_1 \cap b_0 v_0 \notin (\overline{b_0 v_0} \cup \overline{b_1 v_1})$: in this case, in the subsequent activations, b_1 and b_0 will set their colors to *blue*; (ii) $b_1 \in b_0 v_0$: this might not be detected by the local view of b_1, but it is detected by b_0 that sets its color to *orange* ; in the next activations b_1 will move so to transform b_0 into a corner and, after this move, an activation of b_0 will set $s_0 = blue$. So, in both sub-cases we eventually reach a configuration in which b_1 and b_0 are *blue* corner-agents. In the subsequent activations, they will set their color to *red* , proving the lemma. $\square_{Lemma\ 7}$

Lemma 8. *Given a configuration $\mathcal{C}$, let $e = \overline{v_0 v_1}$ be an edge with $q > 2$ edge-agents on it. Eventually all these agents will become corners and set their color to* red.

Proof. The two edge-agents $b_0, b_1 \in e$ that are neighbors of *red* corners, execute the same code described in the previous lemma. So, they wil reach a configuration $\mathcal{C}_{r'}$ in which b_0 and b_1 are *blue* corner-agents. In this case, they wait until all the agents on e move on the segment $\overline{b_0 b_1}$; then, they set their color to *red* (see the rule 3 of Blue Case). It is straightforward to see that each remaining agent on e will move now towards this new edge without colliding, since all movements to the same edge are on parallels trajectories. It follows that, in finite time, a new edge e' is formed with $q - 2$ agents. Iterating the reasoning we will end up in a case where the number of edge-agents on the same edge is at most 2, hence, by Lemmas 5-7, the lemma follows. $\square_{Lemma\ 8}$

Theorem 2. *The* COMPLETE VISIBILITY *problem is solvable in SSYNC by a team of oblivious, obstructable agents, using five colors without creating any collision.*

Proof. From Theorem 1 we have that from any configuration $\mathcal{C}_0$ we reach a configuration $\mathcal{C}_{ID}$ where $\mathcal{I}_{ID} = \emptyset$. This is locally detected by agents (see Lemma 2), that start executing the ED phase. By Lemma 4 we have that the number of *red* corners is not decreasing during the execution of the algorithm. From Lemmas 5-8 we have that eventually each edge-agent a of $\mathcal{H}_{ID}$ will became a *red* corner. So we will reach a configuration $\mathcal{C}_{final}$ in which all agents are corner of $\mathcal{H}_{final}$, thus, they cannot obstruct each other. Moreover, It is easy to see that each agent is able to detect not only local termination, when it sets its color to *red*, but also global termination of ED phase, and thus of the algorithm, when each agent in its local view is *red*. $\square_{Theorem\ 2}$

4 Complete Visibility in ASYNC

In this section we consider the asynchronous model (ASYNC), where there is no common notion of time or rounds, there are no assumptions on time, on activation, on synchronization; moreover, each Compute and Move operation and inactivity may take ant unpredictable (but finite) amount of time, unknown to the agent. As a consequence, agents can be seen while moving, and their computations and movements may be based on obsolete information.

Asynchronous Interior Depletion phase. The INTERIOR DEPLETION algorithm of Sec. 3.1 works also in ASYNC without modifications. We only need to show that the asynchronous behaviour of the agents, and in particular the asynchronous assignment of colors, cannot induce a collision among interior-agents. Since agents always move perpendicularly to the closest edge, it is easy to see that this does not happen and thus Lemmas 1 and Theorem 1 hold also in the asynchronous case.

Asynchronous Edge Depletion phase. The Edge Depletion phase has to be modified for ASYNC. To see why the EDGE DEPLETION algorithm would not work, consider, for example, the Yellow Case in Algorithm 3: it is possible that a moving *yellow* agent is seen by another *yellow* agent, this could lead to scenarios in which an agent assumes color *red* while it is on the edge of the convex hull and not on a corner.

The source of inconsistencies is the fact that agents can be seen while in transit. To prevent this problem we use new colors (*yellow_moving* and *blue_moving*) to signal

that the agents are in transit; those agents will take color *yellow* (resp. *blue*) once as the movement is completed. Using these intermediate colors, we can simulate the ED phase of the previous Section (for $|\mathcal{V}_0| > 2$).

More precisely, in the *Edge Depletion* algorithm of Figure 3, instead of becoming *yellow*, a *brown* agent becomes *yellow_moving*, turning *yellow* at the next activation. Similarly, instead of becoming *blue*, a *yellow* agent becomes *blue_moving*, turning *blue* only when seeing that the "companion" agent is *blue_moving* or *blue*.

It is not difficult to see that, with these additional colors, since agents will always move inside the safe zones of $\mathcal{H}$, the validity of Lemmas 4, 7-8 holds also in ASYNC.

The case of $|\mathcal{V}_0| = 2$. When the agents initially form a line, the algorithm described for SSYNC where the agents first move to a configuration $|\mathcal{V}_0| > 2$, and then apply the general Algorithm, would not work. Consider, for example, the following scenario: both extreme agents compute and their destination is in opposite direction, but only one of them actually moves. At this point, the agents on the line set their color to *red* or *brown* , but they will became interior-agents as soon as the slower extreme agent moves from the line towards its destination, thus changing the convex hull.

The idea is to use a completely different algorithm in ASYNC when the initial configuration is a line (refer to Figure 5b). Two additional special colors (*line-extreme* and *line-moving*) are used. The color *line-extreme* is taken by the two agents a_1 and a_2 located at the extreme points of the line, x_1 and x_n, when activated; this color is used to acknowledge the line condition, and to define the smallest enclosing circle SEC with diameter $\overline{x_1, x_n}$. Notice that, due to obstructed visibility, the diameter, and thus SEC, is unknown to the agents. The two extreme agents will never move.

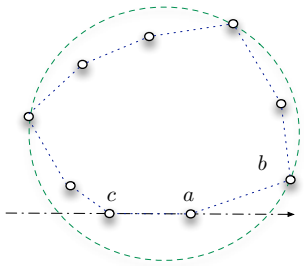
The general strategy is to have the other agents move to points on SEC. First notice that an agent a can detect that the configuration is a line, either by geometric conditions (i.e., it sees only one or two collinear agents), or by the special color of some visible agents (*line-extreme* or *line-moving*). If an uncolred agent a located in x sees a *line-extreme* agent (say a_1), then a changes its color to *line-moving* and it moves perpendicularly to xx_1 toward the perimeter of the circle whose diameter is identified by a_1 and the closest agent $b \neq a_1$ on the line xx_1 (note that there must be at least one, possibly the other extreme). A *line moving* agent follows similar rules; if it can detect SEC (e.g., it sees two *line-extreme*) it continues its perpendicular move towards it. Otherwise, it does not move. It can be shown that, at any time, there is at least one agent that, if activated, can move. A non extreme agent switches its color to *red* when it sees only agents on the SEC; an extreme agents switches its color to *red* when it sees only *red* or *line-extreme* agents.

It is not difficult to see that this set of rules will allow the agents to reach SEC in finite time becoming *red*, and thus to solve the COMPLETE VISIBILITY problem.

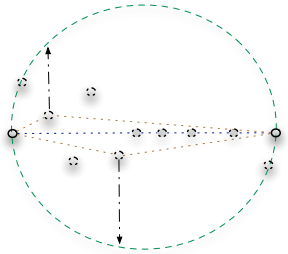
Theorem 3. *The* COMPLETE VISIBILITY *problem is solvable in ASYNC by a team of oblivious, obstructable agents, using eight colors without creating any collision.*

5 Circle Formation in ASYNC

When executing the previous algorithm, the agents reach a configuration $\mathcal{C}_{final}$ in which all agents are corners of $\mathcal{H}_{final}$. Starting from this particular configuration it is possible to arrange the agents in such a way to reach a configuration $\mathcal{C}_{circ}$ in which each agent is positioned on the $SEC(\mathcal{C}_{final})$. Note that the solution of the COMPLETE VISIBILITY problem when $|\mathcal{V}_0| = 2$ already form a circle, hence we focus on the case $|\mathcal{V}_0| > 2$.



(a) CIRCLE FORMATION: Agent a is neighbor of an agent b on SEC, so it moves on line ca in direction of SEC. During this movement, the corner-agents of the convex hull are not modified, and visibility with the other agents is preserved.



(b) ASYNCH FORMATION FOR $|\mathcal{V}_0| = 2$: the two extreme agents signal the line configuration with color *line-extreme*, the other agents move perpendicularly to them until they reach the SEC whose diameter is defined by the extreme agents.

Notice that, when all agents are on ∂H_{final} they can compute the same $SEC(\mathcal{C}_{final})$ since all the local views are consistent. Moreover, there exists a set of agents $\mathcal{X} \subseteq \mathcal{A}$ that are already on SEC, and $|\mathcal{X}| \geq 2$. The idea of the algorithm is to move all agents on SEC in such a way that in each point of their trajectories they can see a subset of nodes $\mathcal{Y}$ such that $SEC(\mathcal{Y}) = SEC(\mathcal{X}) = SEC(\mathcal{C}_{final})$. More precisely, the moving rule allows agents to move towards SEC if they are "neighbors" (i.e., neighboring corner) of some agent on SEC in $\mathcal{C}_{final}$ (see Figure 5a). Let a be neighbor of some b already on SEC, and let c be its other neighbor: a will move toward SEC on line ca guaranteeing that the corner-agents of the convex hull stay corner-agents, and do not loose visibility with any other agent. Note that, unless in final position, there is always at least one agent that can move. The algorithm terminates when all the agents are on SEC. It is not difficult to see that:

Theorem 4. *Starting from a configuration $\mathcal{C}_{final}$ in which all the agents are corners, there is an algorithm in ASYNC that makes the agents reach a configuration $\mathcal{C}_{circ}$ in which each agent occupies a different position on $SEC(\mathcal{C}_{final})$ without colliding.*

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