

Complexity of Critter Crunch

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Critter Crunch



Rules of Critter Crunch

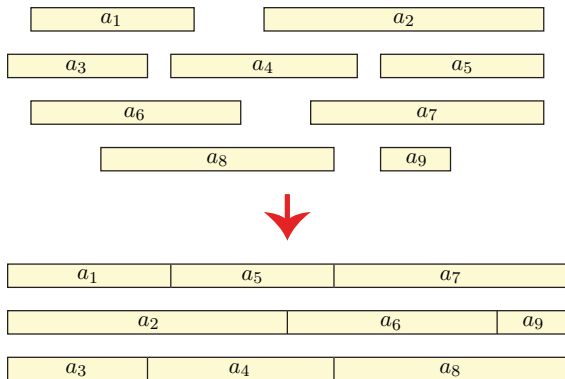
- Critters come in different sizes and colors.
- The player can pick up the bottom Critter of any column and shoot it into any column in order to rearrange or feed Critters.
- A Small Critter can be fed to a Medium one (but not to a Large one), and a Medium Critter can be fed to a Large one.
- When a Critter eats another one, it becomes full. As soon as a Critter eats two empty Critters or one full Critter, it explodes.
- When a Critter explodes, all adjacent Critters of the same size and color explode as well (and so on, recursively).
- When a Critter has an empty space above, it climbs up.
- When a Critter eats or collides with another, it may be eaten by the Critter above it (if their sizes are appropriate).
- A Blocker Critter cannot be picked up by the player, cannot eat, and can be eliminated only if there are no other Critters below it, by popping the Critter immediately above it.

Recall: 3-Partition is strongly NP-complete

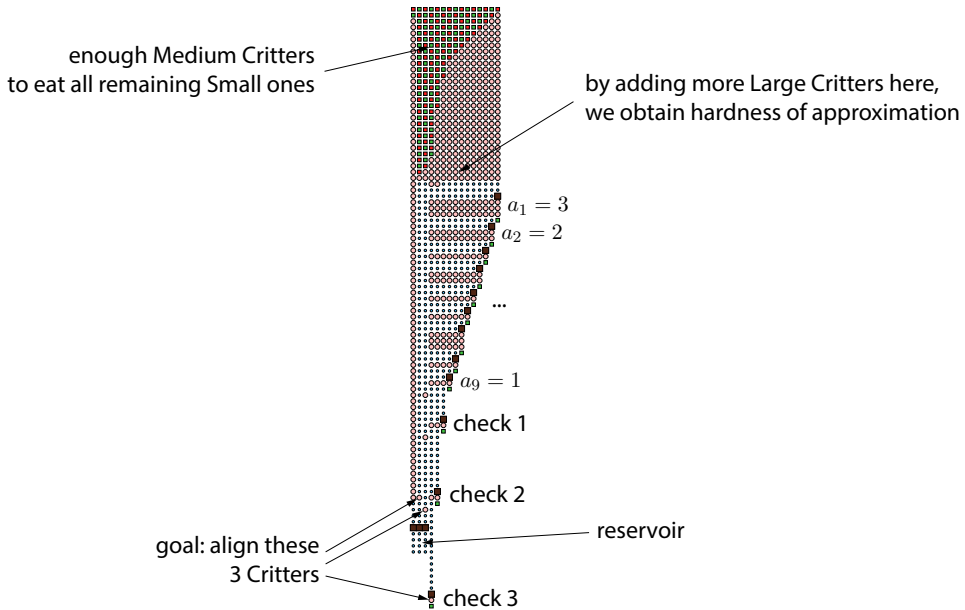
3-Partition problem:

Input: positive integers a_1, a_2, \dots, a_n with $B/4 < a_i < B/2$.

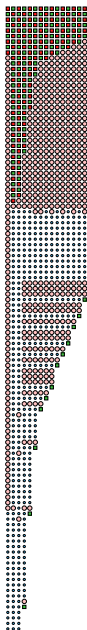
Output: YES if there is a partition of the n integers into subsets each of which has sum B . NO otherwise.



NP-hardness with Blocker Critters



NP-hardness with limited number of moves (puzzle mode)



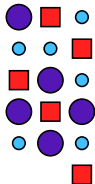
goal: eliminate every Critter
in 12 moves ($n + n/3$)

The following results hold for Critter Crunch levels with no limits on the number of moves:

- No Blocker Critters, no Large Critters, but many colors
 - 1-column levels: solvable in $O(n)$ (trivially)
 - 2-column levels: NP-complete (reduction from Vertex Cover)
 - 3-or-more-column levels: solvable in $O(n)$
- No Blocker Critters and only 1 color per size
 - 1-column levels: solvable in $O(n)$ (trivially)
 - 2-column levels: NP-complete (reduction from 3-Partition)
 - 3-or-more-column levels: solvable in $O(n)$

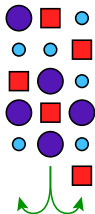
3 or more columns: sorting Critters

Lemma 1: it is possible to rearrange Critters so that each column has Critters of only one size without feeding any Critter.



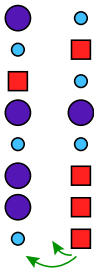
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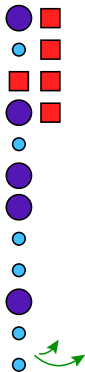
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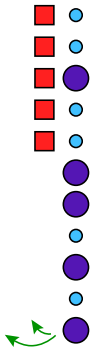
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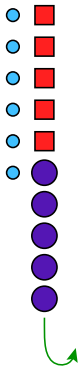
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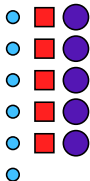
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Primitive explosions

All the moves that cause Critters to explode can be expressed as sums of the following “primitive” ones:

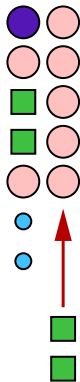
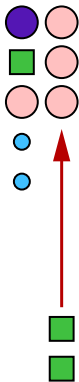
Small	Medium	Large
2	1+	
3	2+	
	2	1+
	3	2+
1	1	1+
1	2	1+

In addition, there are explosions that can only occur as part of a combo:

Small	Medium	Large
2	2	1
2	1	1+

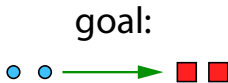
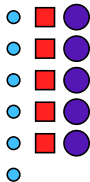
Combo-only explosion: example

For example, the following moves eliminate all Critters:



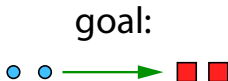
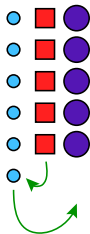
Setting up primitive explosions

Lemma 2: all non-combo primitive explosions can be replicated from the sorted position given by Lemma 1.



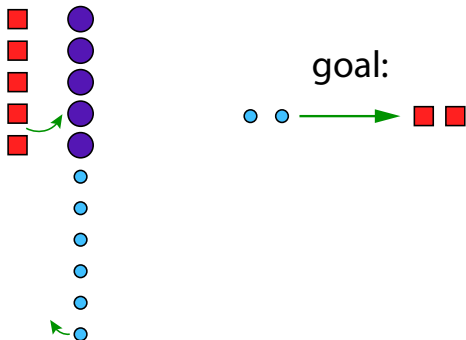
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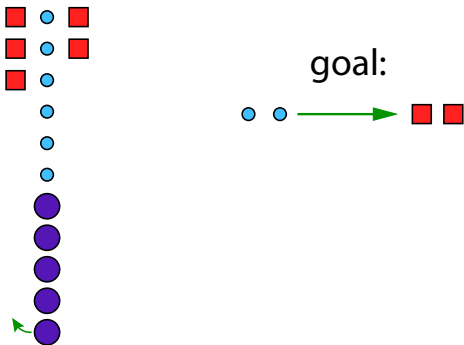
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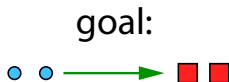
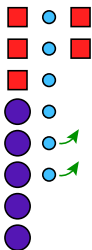
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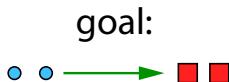
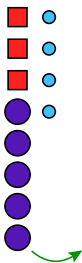
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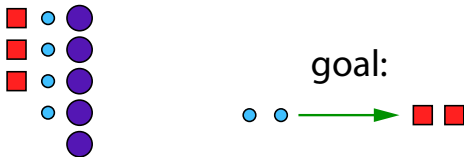
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No Large Critters and 3 or more columns

Theorem 1: if there are 3 or more columns, s Small Critters, m Medium Critters of c different colors, and no Large Critters, then all Critters can be eliminated if and only if $2c \leq s \leq 2m$.

Proof:

- At least 2 Small Critters must be used to trigger an explosion, and each explosion only affects Critters of one color.
 $\implies 2c \leq s$.
- Each Medium Critter can eat at most 2 Small ones.
 $\implies s \leq 2m$.
- Whenever $2c \leq s \leq 2m$, we can set up a primitive explosion that eliminates some Critters and preserves both inequalities.

Example: if $c = 3$, $s = 7$, $m = 8$, we use 3 Small Critters to explode all the Medium Critters of the most numerous color, obtaining $c = 2$, $s = 4$, $2 \leq m \leq 5$.

Only 1 color per size and 3 or more columns

Combo-only explosions can always be converted into sums of non-combo primitive explosions.

Example: suppose that the following combo occurs:

Small	Medium	Large	
1	2	3	(primitive)
2	2	1	(combo-only)

This combo is equivalent to the following sequence of non-combo primitive explosions:

Small	Medium	Large	
1	1	4	(primitive)
2	3		(primitive)

Only 1 color per size and 3 or more columns

Theorem 2: if there are 3 or more columns and s Small Critters, m Medium Critters of only 1 color, and l Large Critters of only 1 color, then all Critters can be eliminated if and only if:

$$\left\{ \begin{array}{ll} 2 \leq m \leq 2l & \text{if } s = 0, \\ 1 \leq m \leq 2l & \text{if } s = 1, \\ (m = 1 \wedge l = 0) \vee (m = 2 \wedge l \neq 1) \vee m \geq 3 & \text{if } s = 2, \\ (s = 2m \wedge l = 0) \vee s < 2m & \text{if } s \geq 3. \end{array} \right.$$

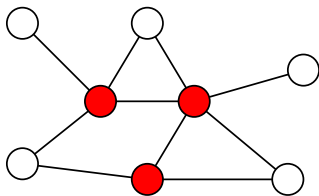
Proof: similar to Theorem 1 but with some case analysis...

Recall: Vertex Cover is NP-complete

Vertex Cover problem:

Input: a graph $G = (V, E)$ and an integer k .

Output: YES if there is a subset $U \subseteq V$ of exactly k vertices such that each edge in E has at least one endpoint in U . NO otherwise.



$$k = 3$$

No Large Critters and only 2 columns: NP-hardness

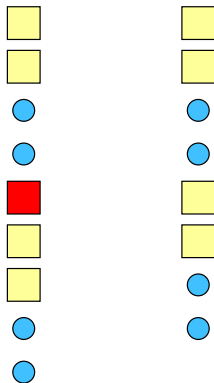
We assign a different color to each vertex of G , and for each edge we use this gadget:



Note that we need 4 extra Small Critters to eliminate all the Medium Critters in the edge gadget.

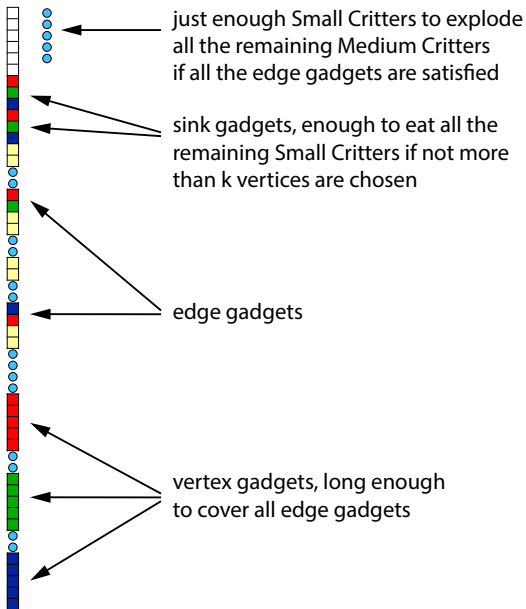
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However, if at least one of the two central Critters is removed (i.e., the edge gadget is “satisfied”), then all remaining Critters can be eliminated without using additional Critters.



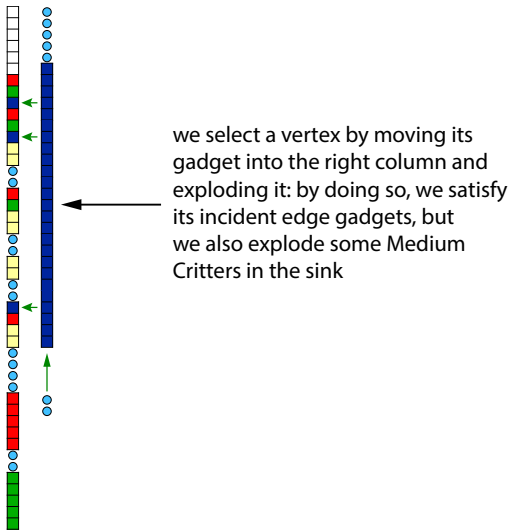
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Full reduction from Vertex Cover (not to scale):



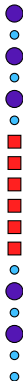
No Large Critters and only 2 columns: NP-hardness

Full reduction from Vertex Cover (not to scale):



Only 1 color per size and only 2 columns: NP-hardness

We use this *shaker gadget*:



It gives us two options:

- 1) skip through it to access other parts of the level
(a limited number of times), or
- 2) eliminate all Critters, in particular exploding a Medium Critter.

Only 1 color per size and only 2 columns: NP-hardness

Sketch of the reduction from 3-Partition (not to scale):

