Super Mario Bros. Is Harder/Easier than We Thought

FUN 2016

Erik D. Demaine, Giovanni Viglietta, Aaron Williams

La Maddalena – June 9, 2016

State of the art: FUN 2014

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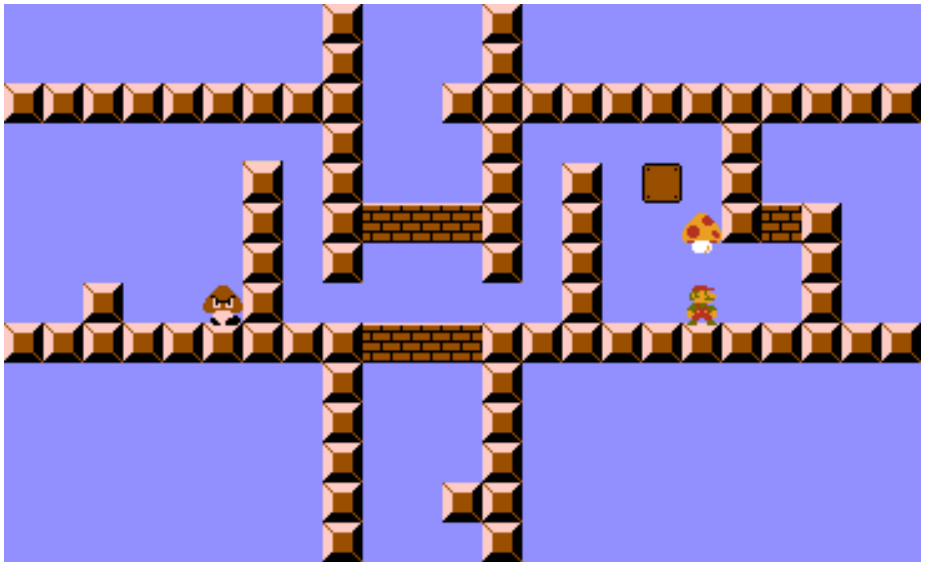

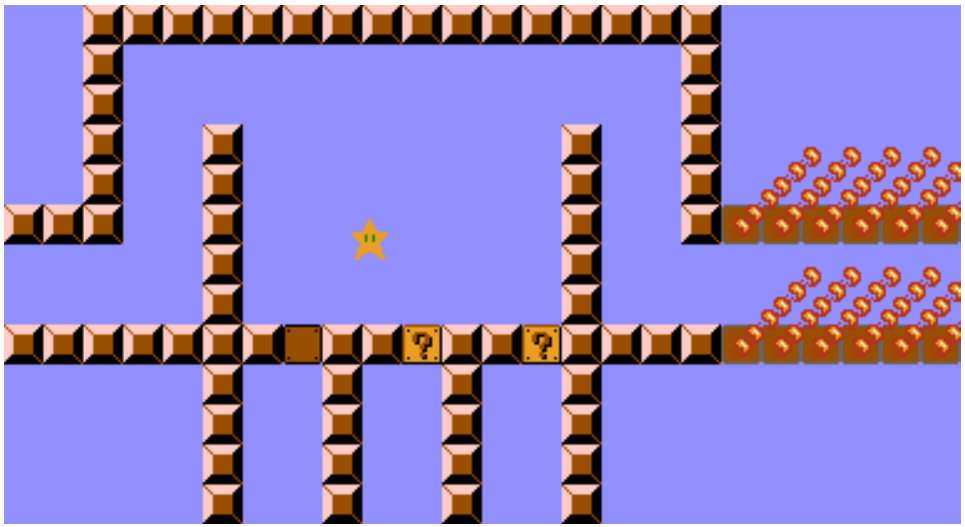

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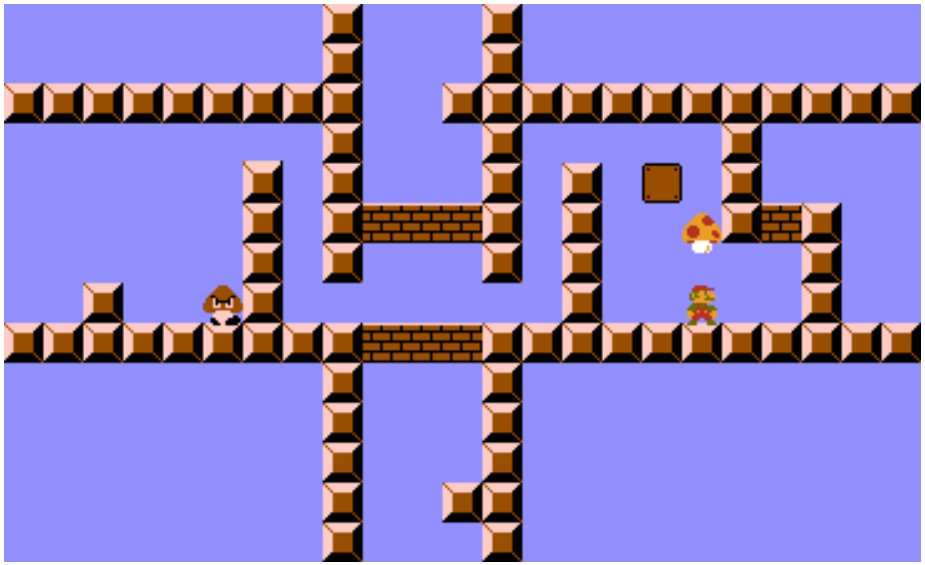

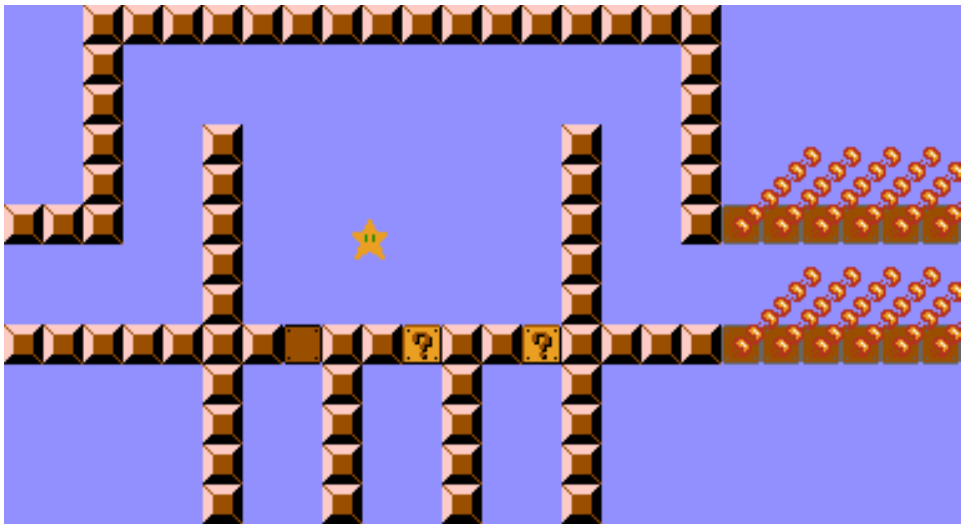

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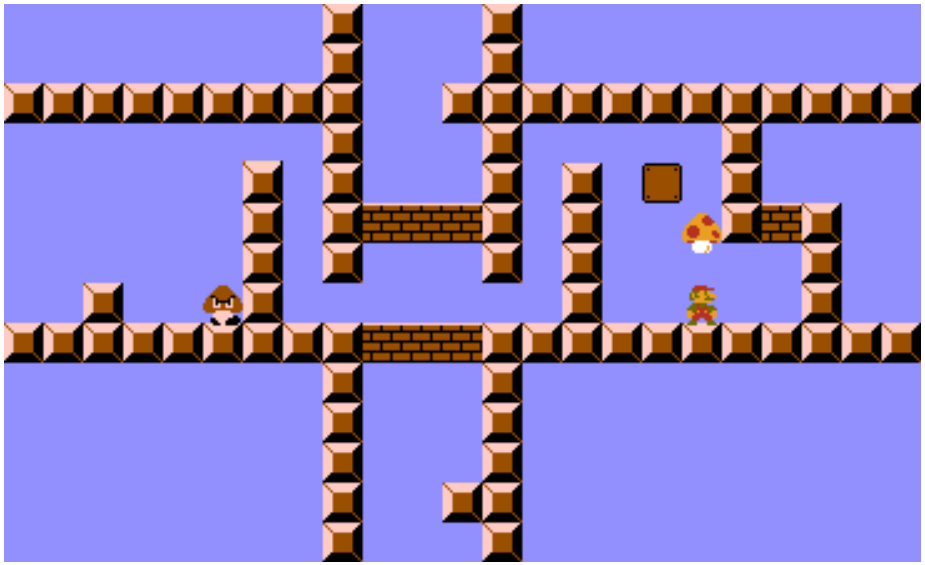

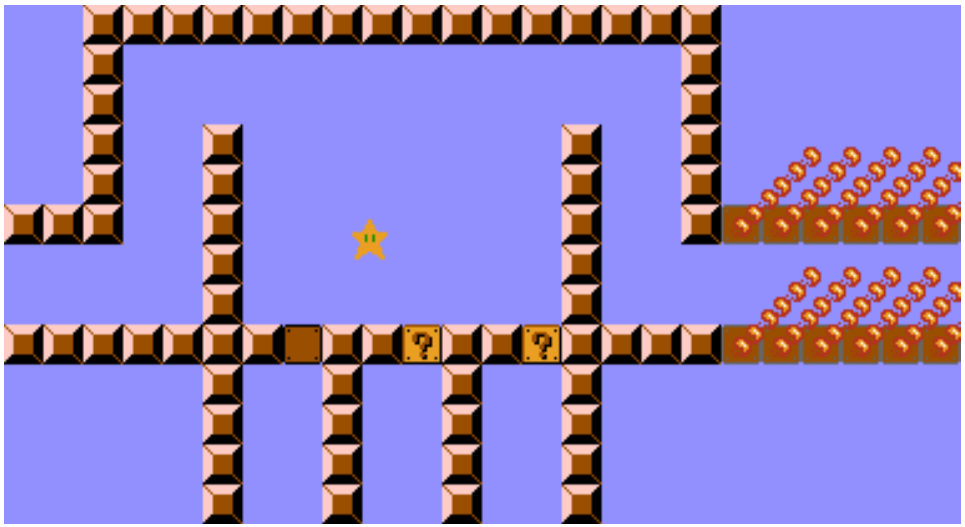

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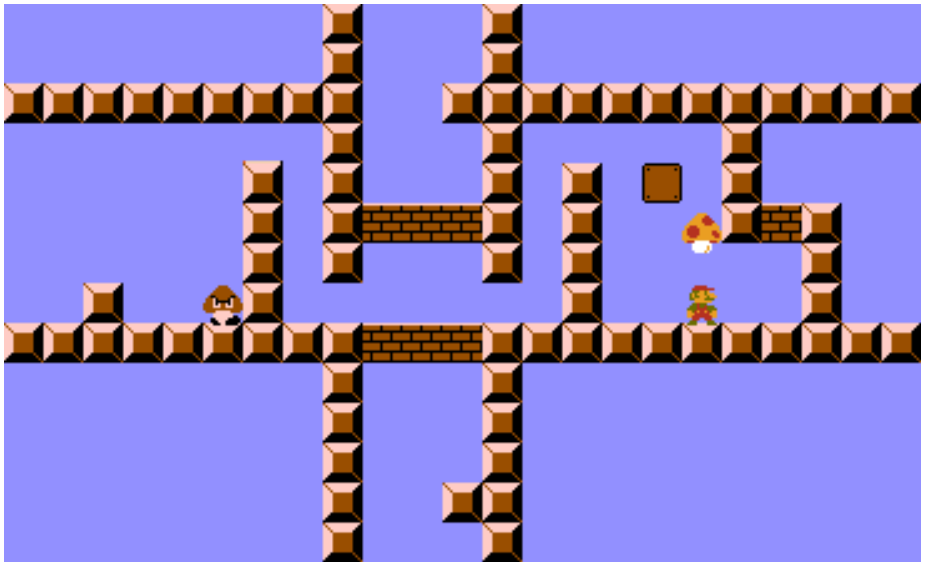

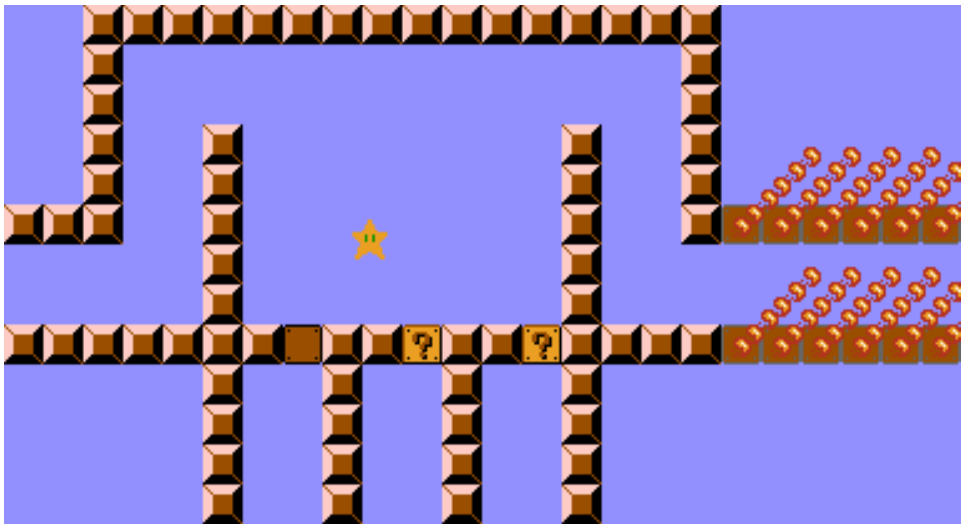

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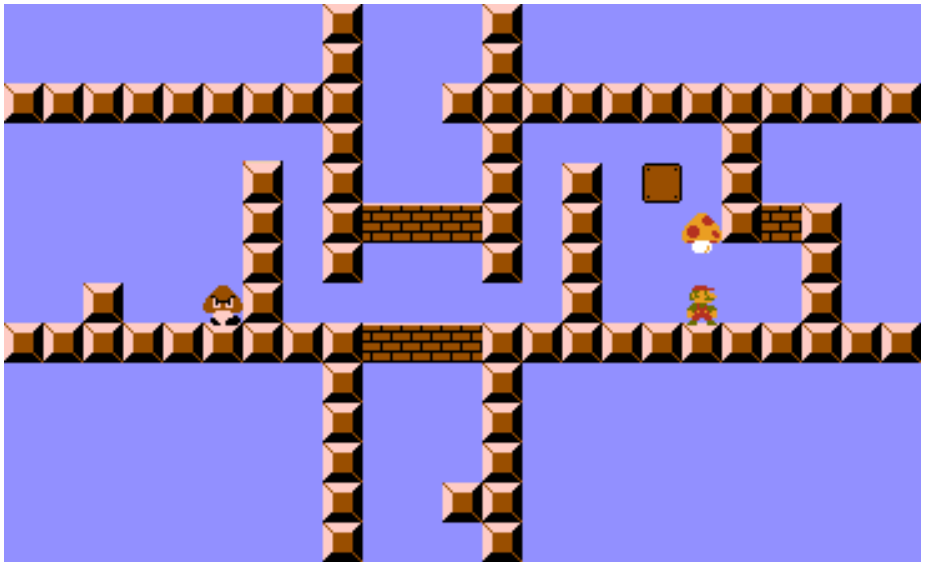

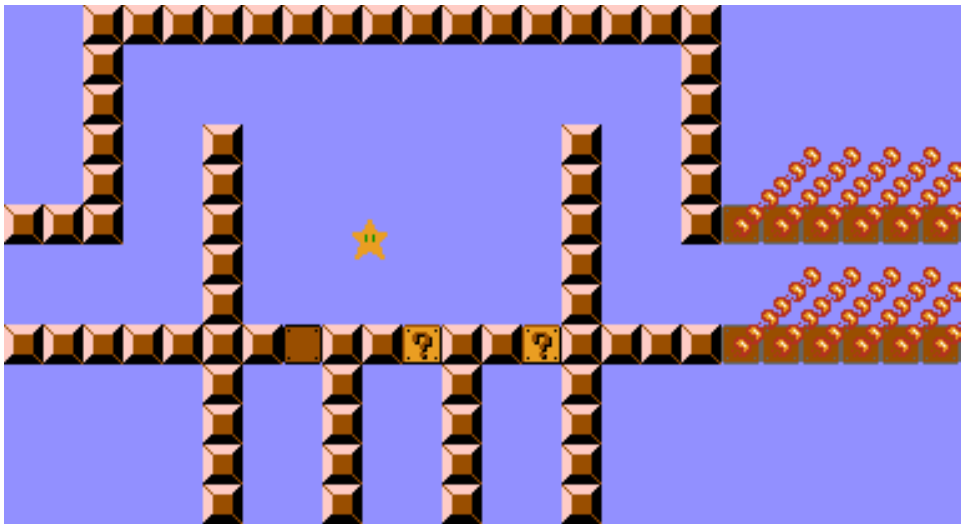

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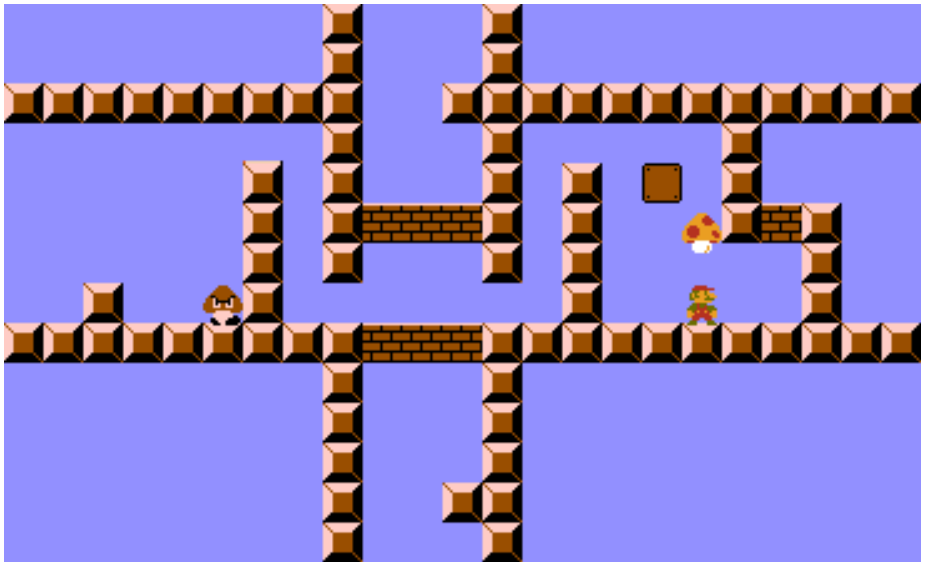

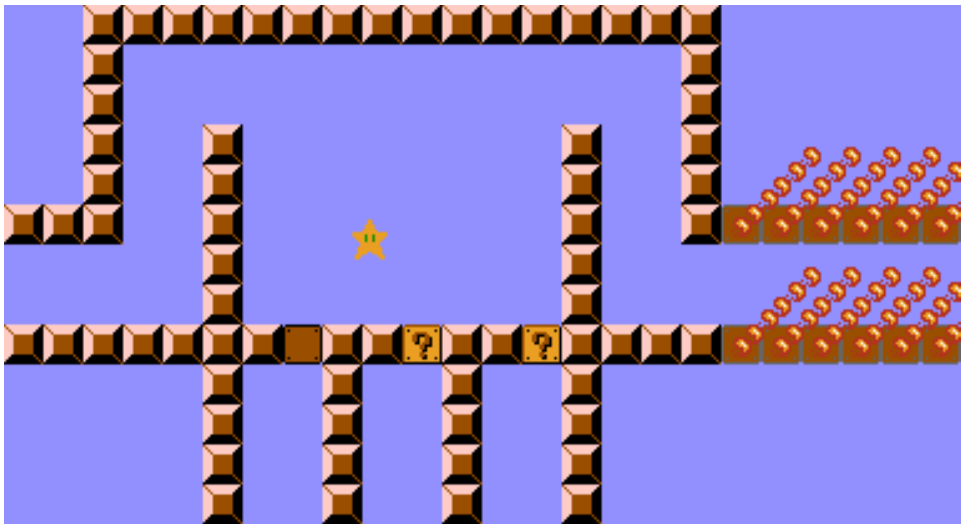

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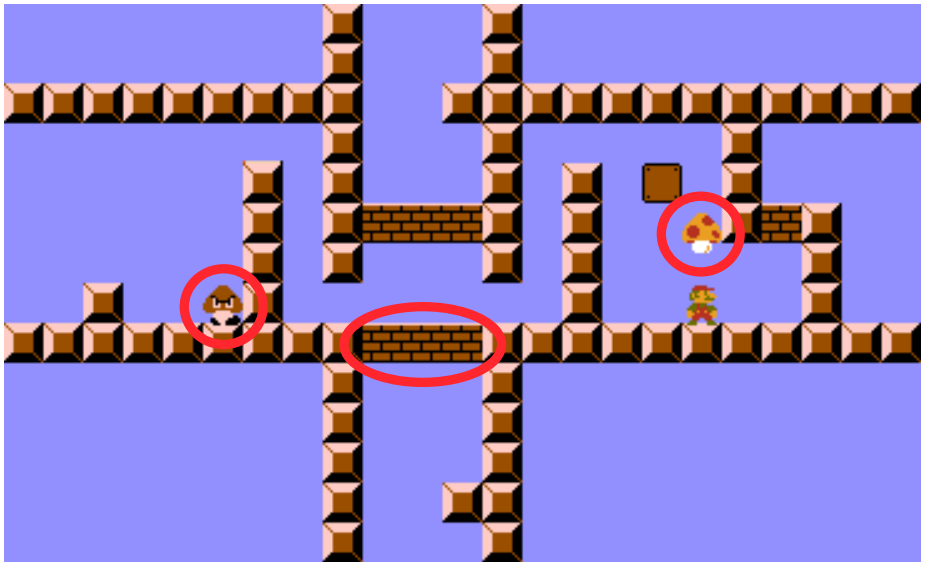

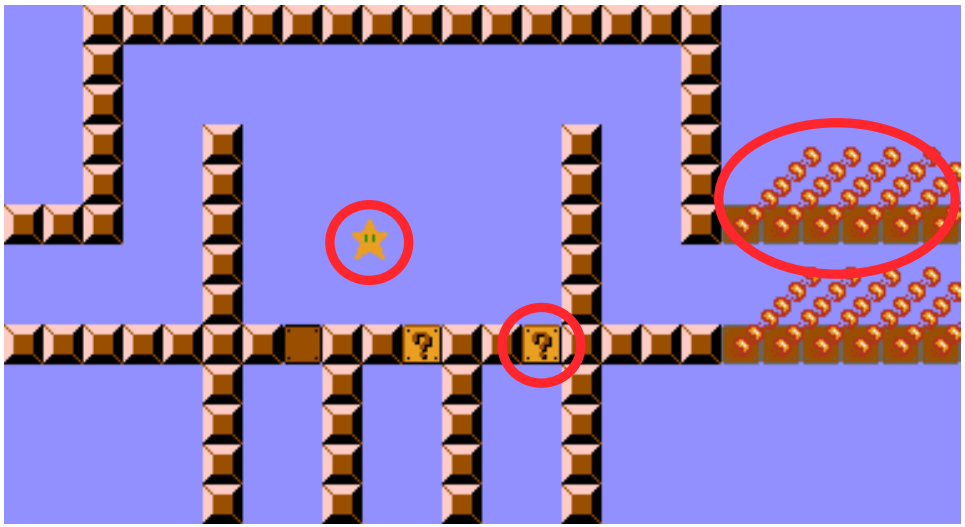

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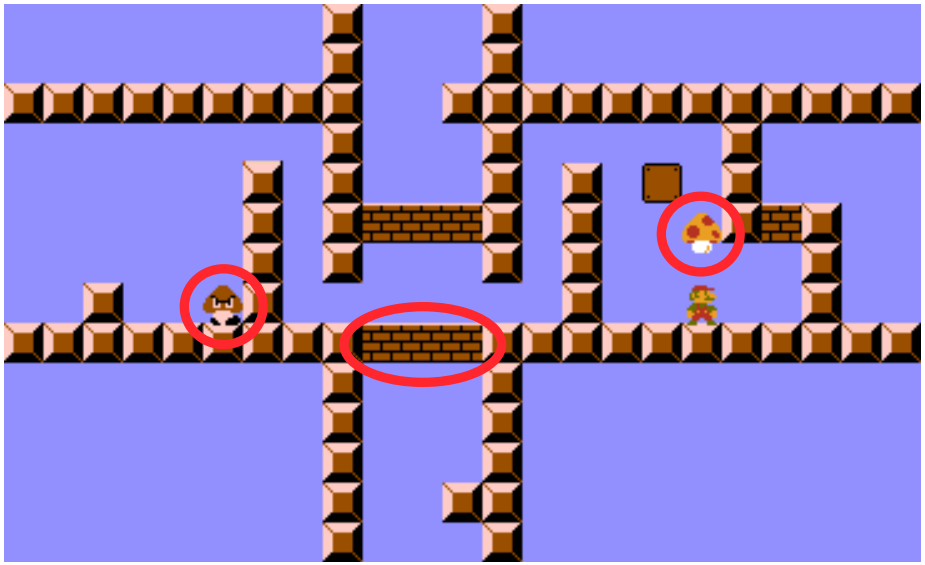

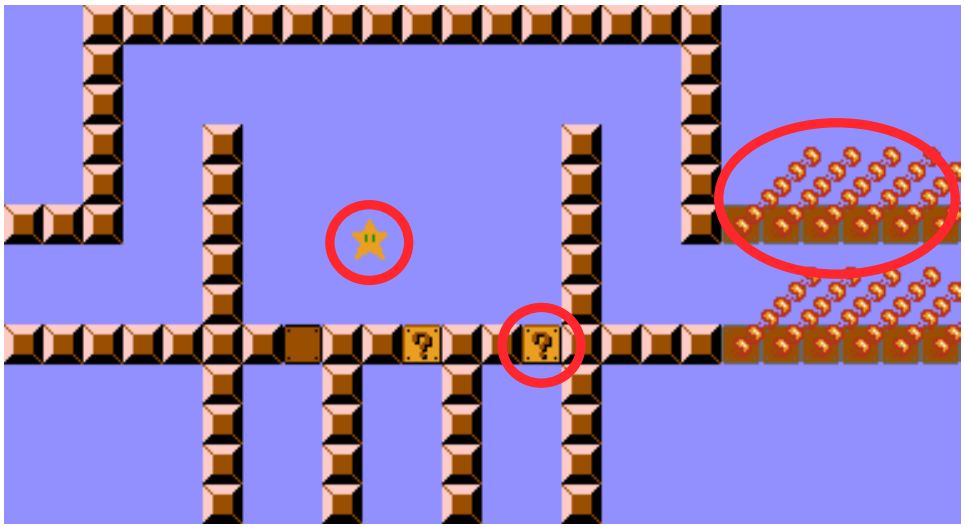

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- Lots of game elements were used
  (we would like to be more parsimonious)
- Levels were not entirely glitch-proof
  (in SMB there are glitches that make
  the old crossover gadget ineffective)

Our results

- Rescuing the princess is in **P** if the standard rules of SMB apply
- Rescuing the princess is weakly **NP-hard** if the level format allows run-length encoding, even if
  - levels have height 2
  - only coins and pipes are used
  - only buttons  and  are used
- Reaching the flagpole in a level without losing lives is **PSPACE-complete** if objects off-screen are not reset

(All constructions are glitch-proof)

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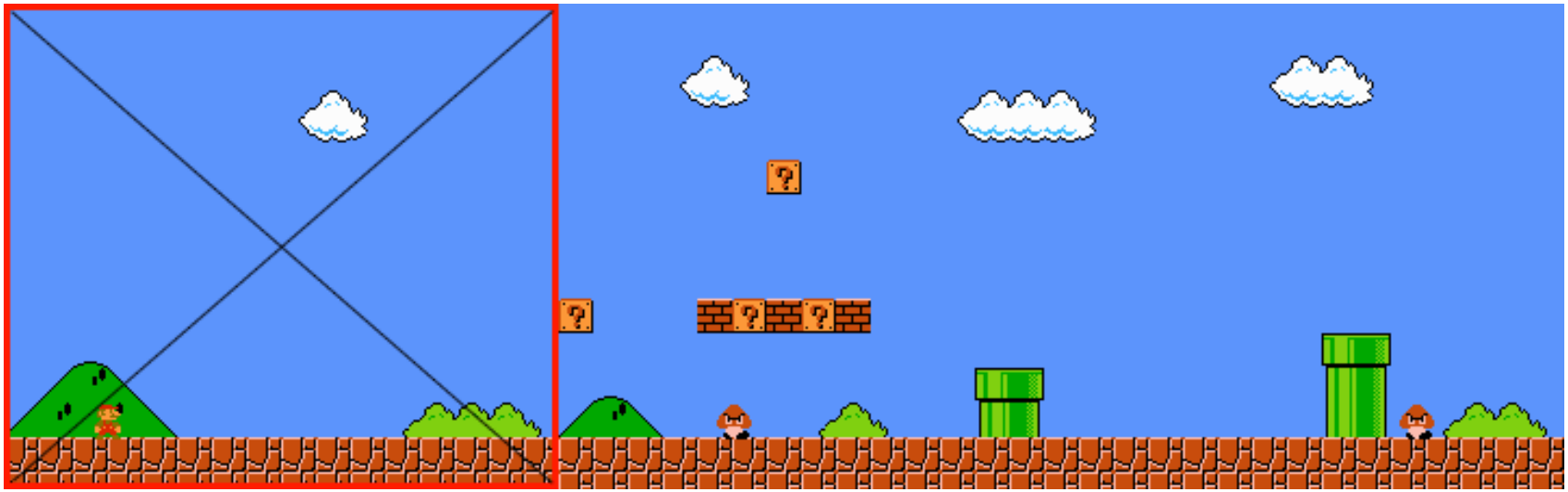

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```
WBootCheck:   lda TopScoreDisplay,x       ;check each score digit in the top score
              cmp #10                     ;to see if we have a valid digit
              bcs ColdBoot                ;if not, give up and proceed with cold boot
              dex
              bpl WBootCheck
              lda WarmBootValidation      ;second checkpoint, check to see if
              cmp #$a5                    ;another location has a specific value
              bne ColdBoot
              ldy #WarmBootOffset         ;if passed both, load warm boot pointer
ColdBoot:     jsr InitializeMemory        ;clear memory using pointer in Y
              sta SND_DELTA_REG+1         ;reset delta counter load register
              sta OperMode                ;reset primary mode of operation
              lda #$a5                    ;set warm boot flag
              sta WarmBootValidation
              sta PseudoRandomBitReg      ;set seed for pseudorandom register
              lda #%00001111
              sta SND_MASTERCTRL_REG      ;enable all sound channels except dmc
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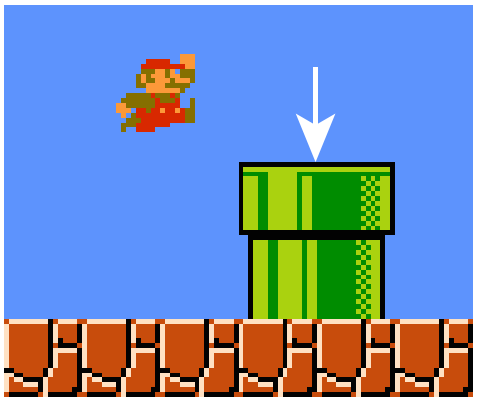

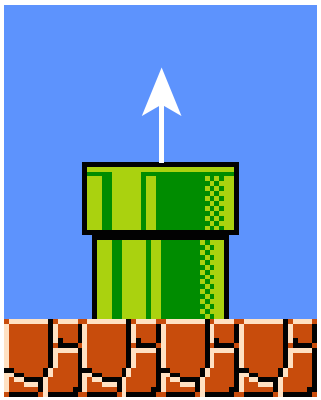

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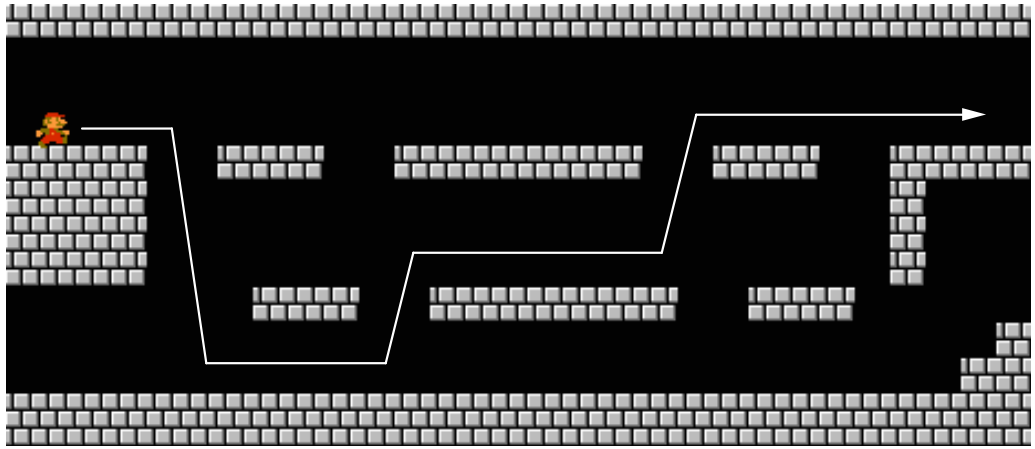

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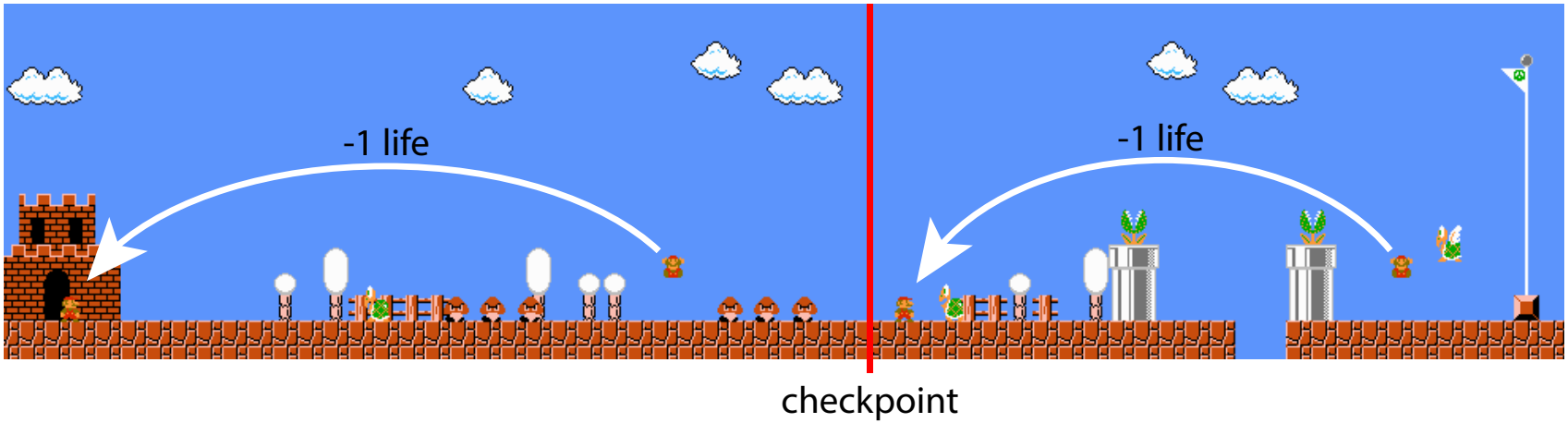

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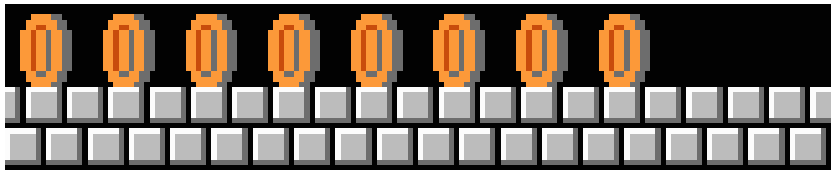
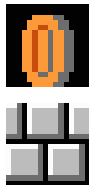
$\Longrightarrow$ Solving a sequence of levels is reduced to exploring a polynomial-size "macro-graph"

Run-length encoding

In SMB's level format, a row of up to 16 equal objects can be encoded as a single object:

the fragment 

can be encoded as  x 8

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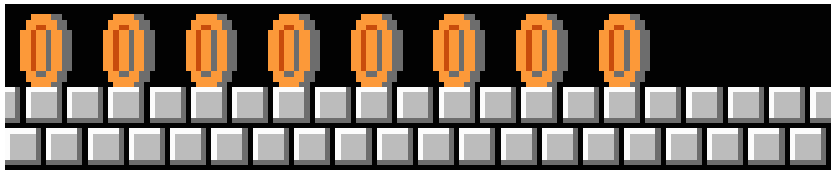
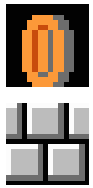
the fragment 

can be encoded as  x 8

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Let us generalize this concept to arbitrarily long rows

NP-hardness: reduction from Knapsack

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The Knapsack problem is weakly NP-complete

$\Downarrow$

SMB with run-length encoding is weakly NP-hard

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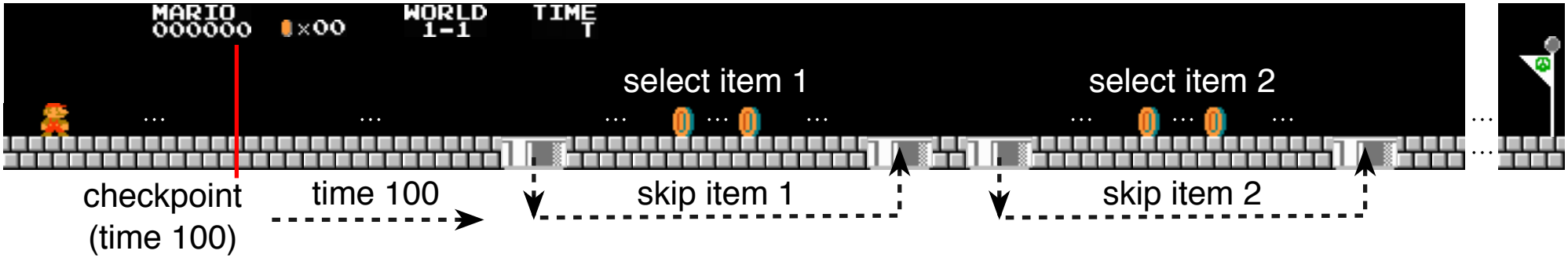

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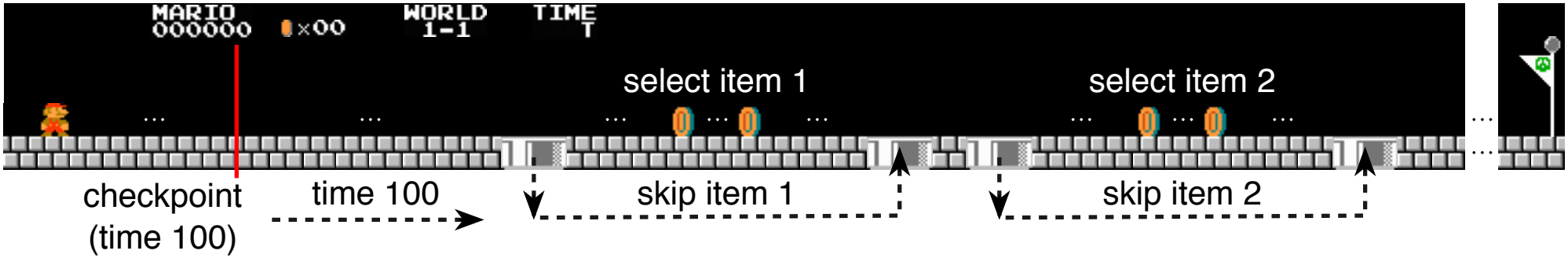

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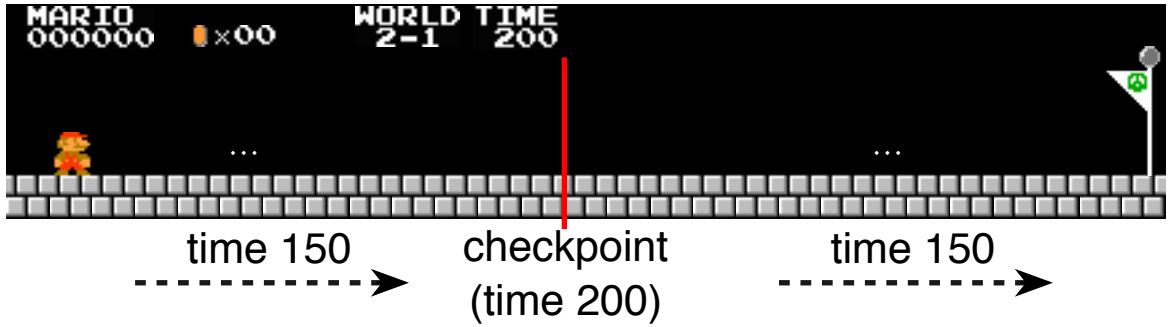

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- **Crossover**, usable multiple times
- **Door**: can be opened, closed, and traversed if and only if it is open

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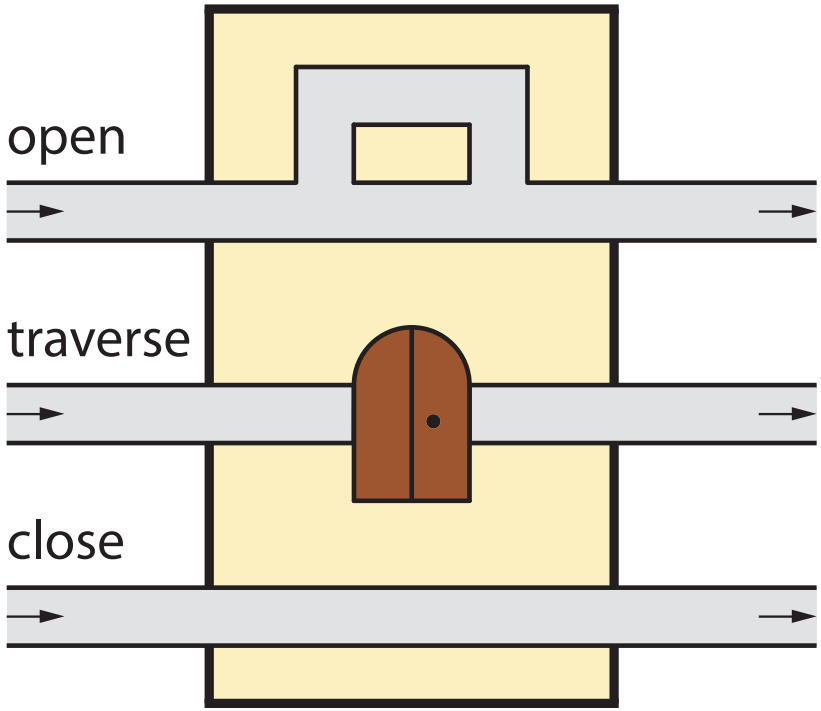

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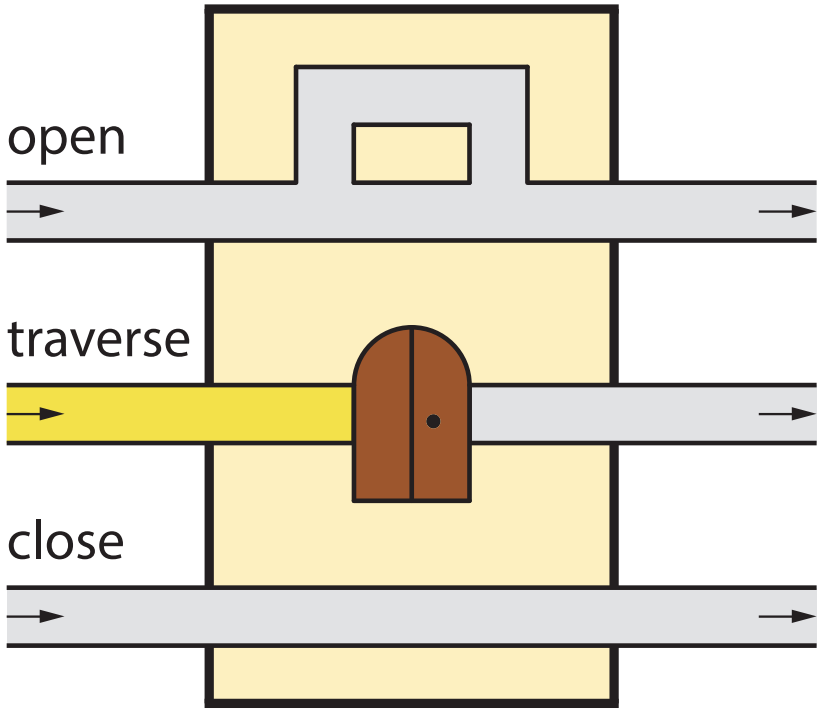

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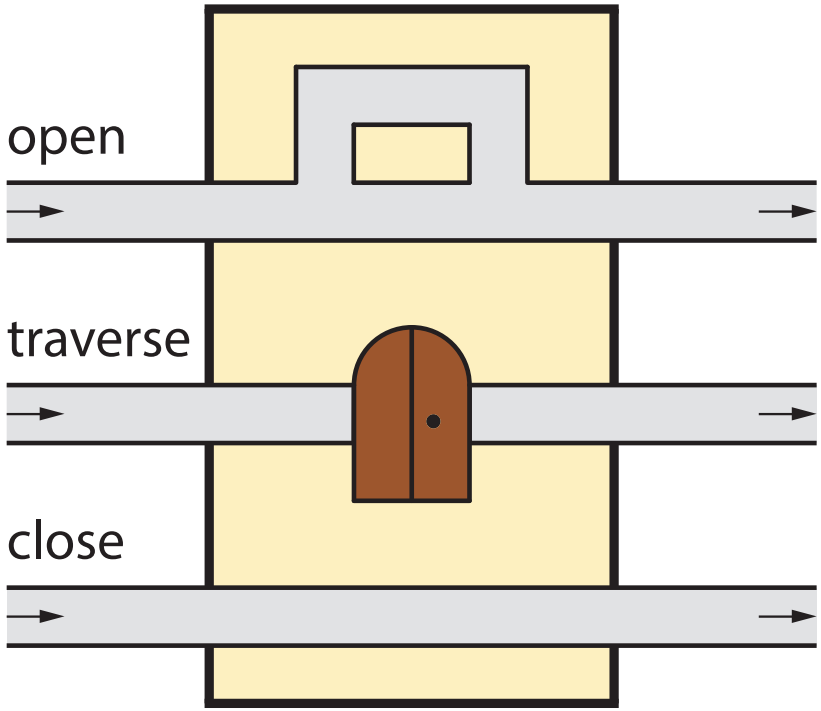

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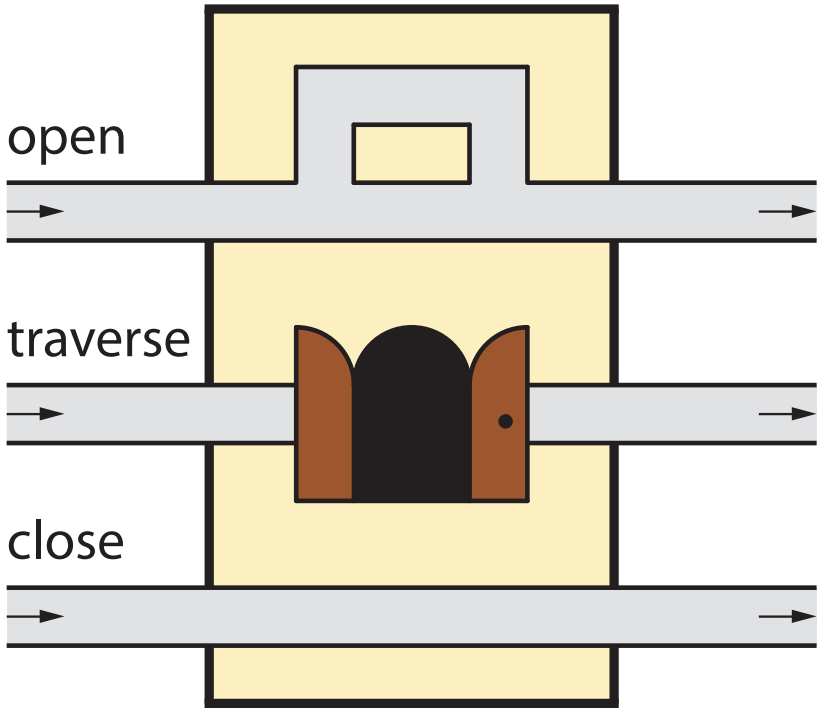

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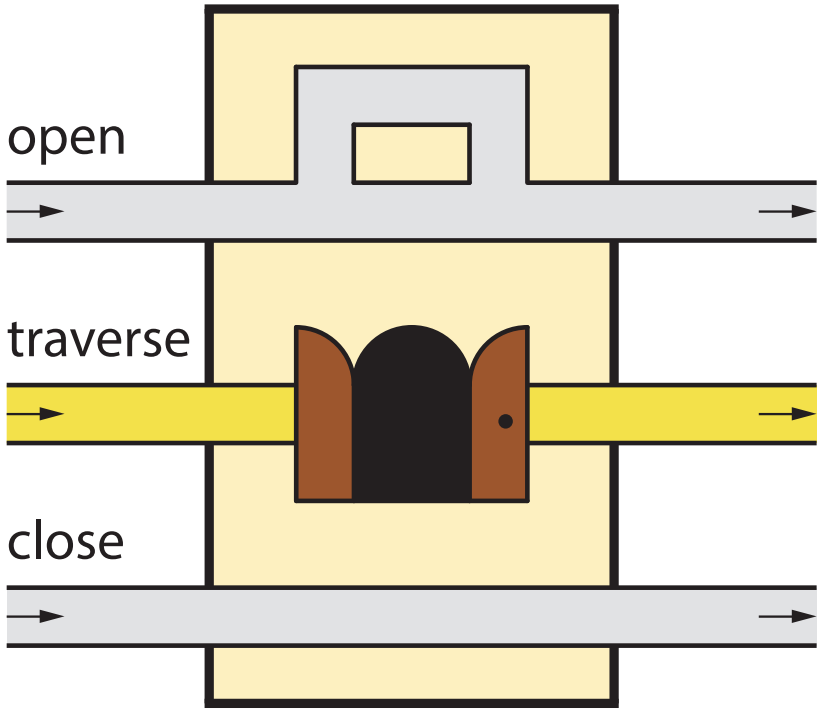

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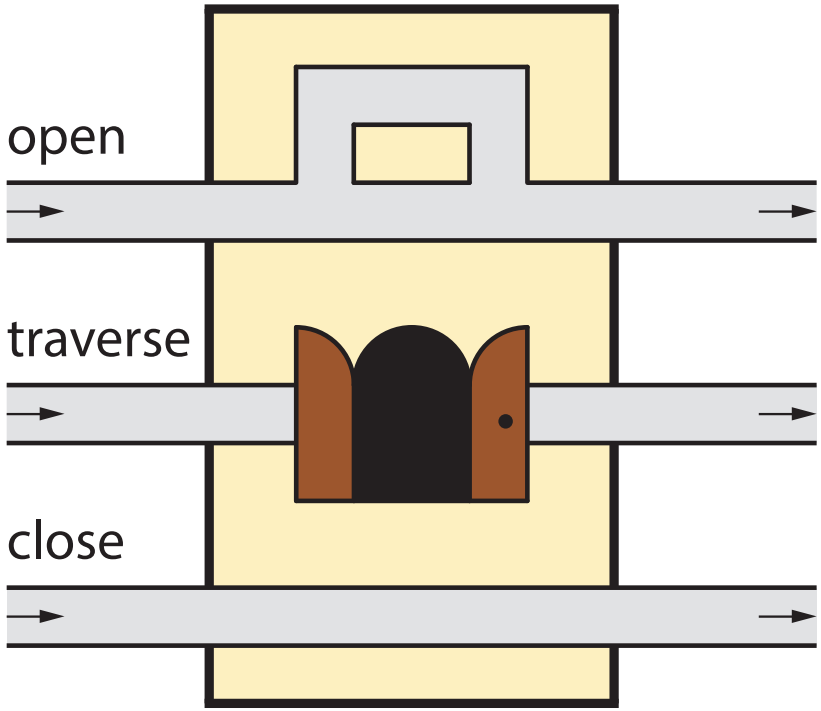

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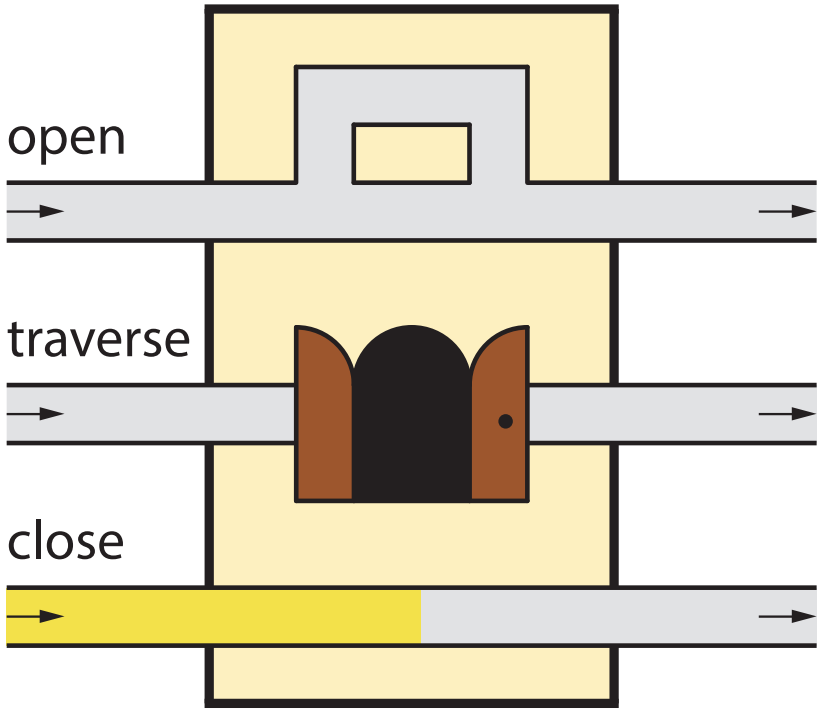

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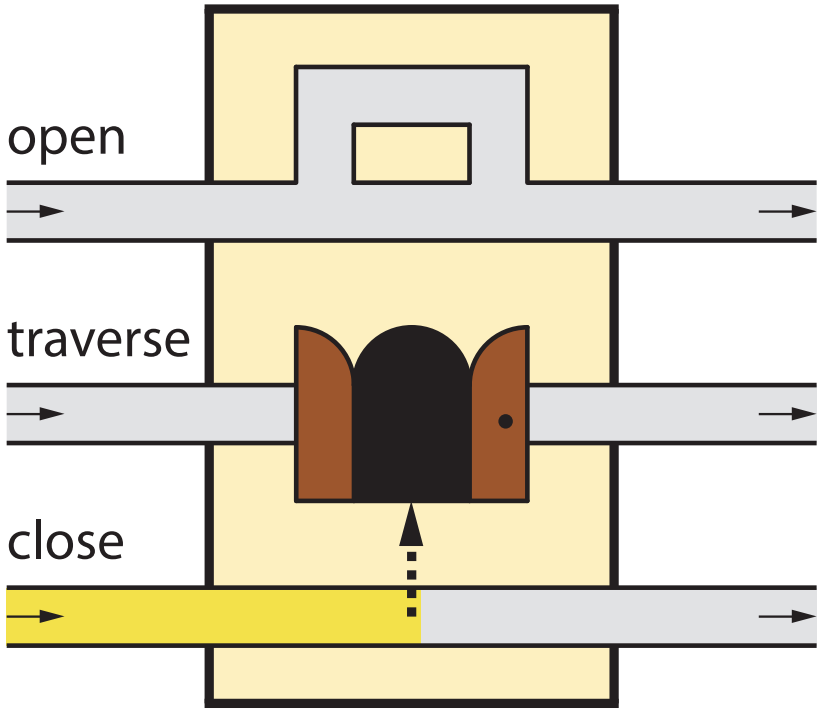

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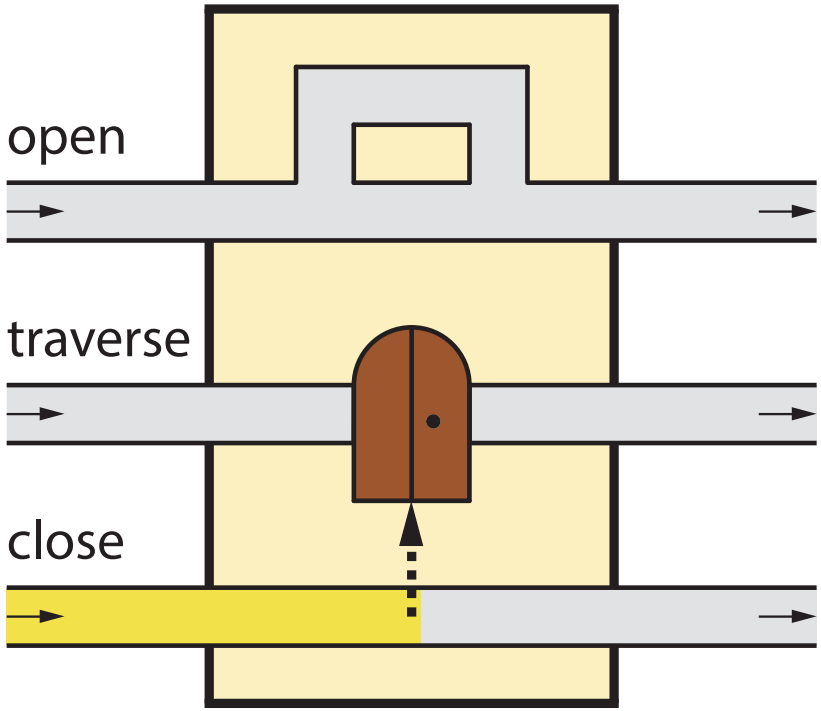

PSPACE-hardness gadgets

- **Start** & **Finish**
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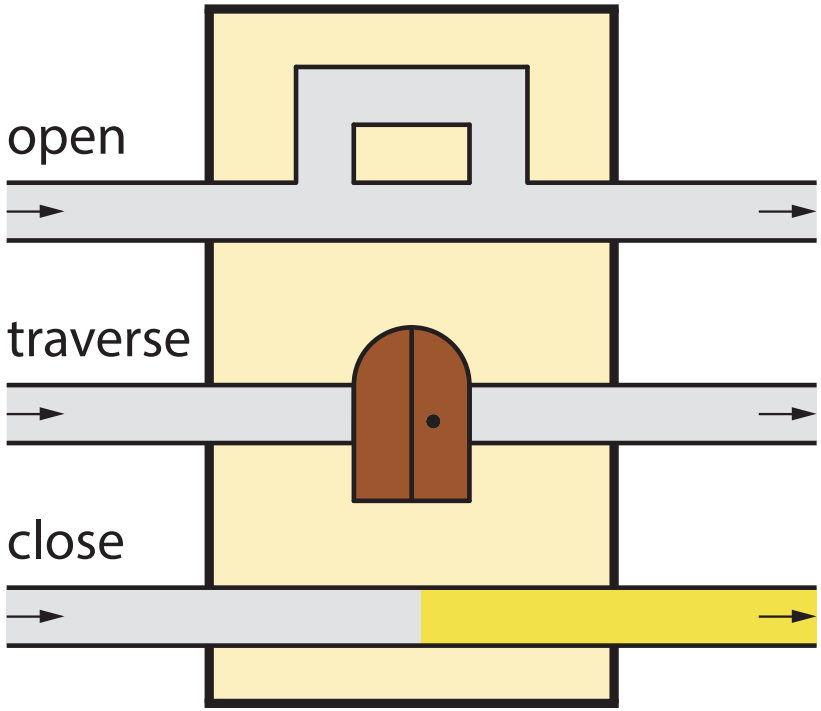

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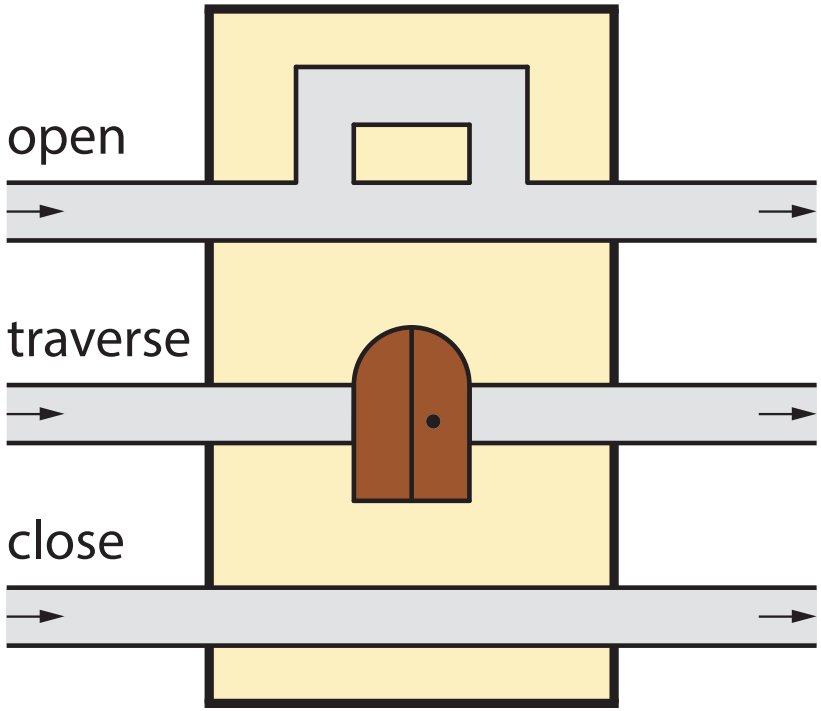

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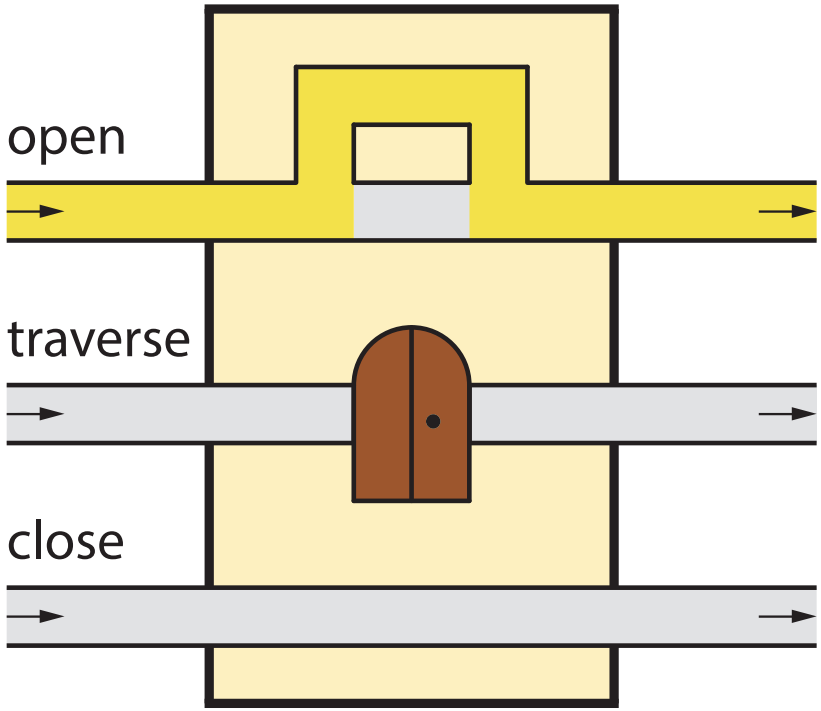

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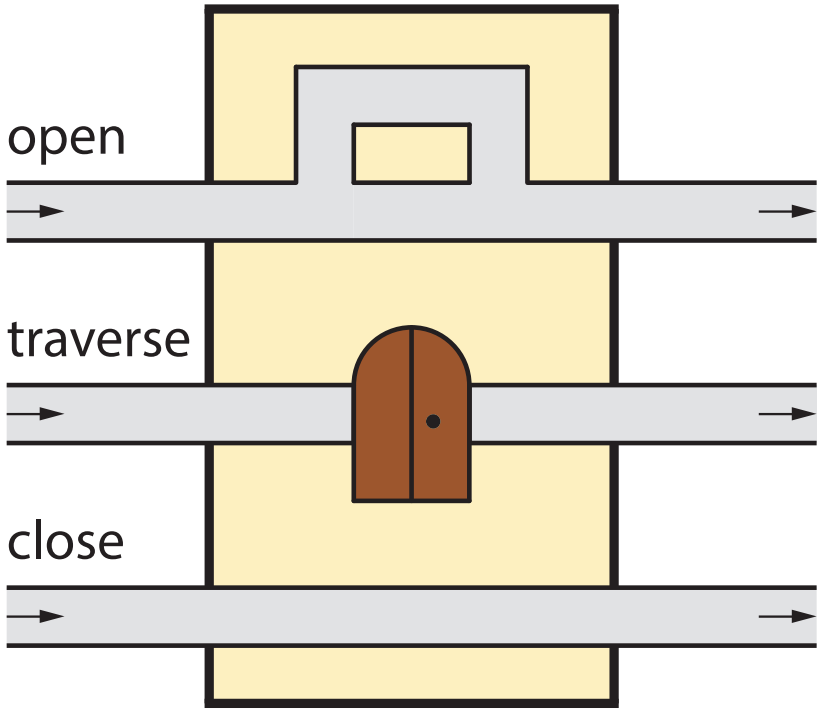

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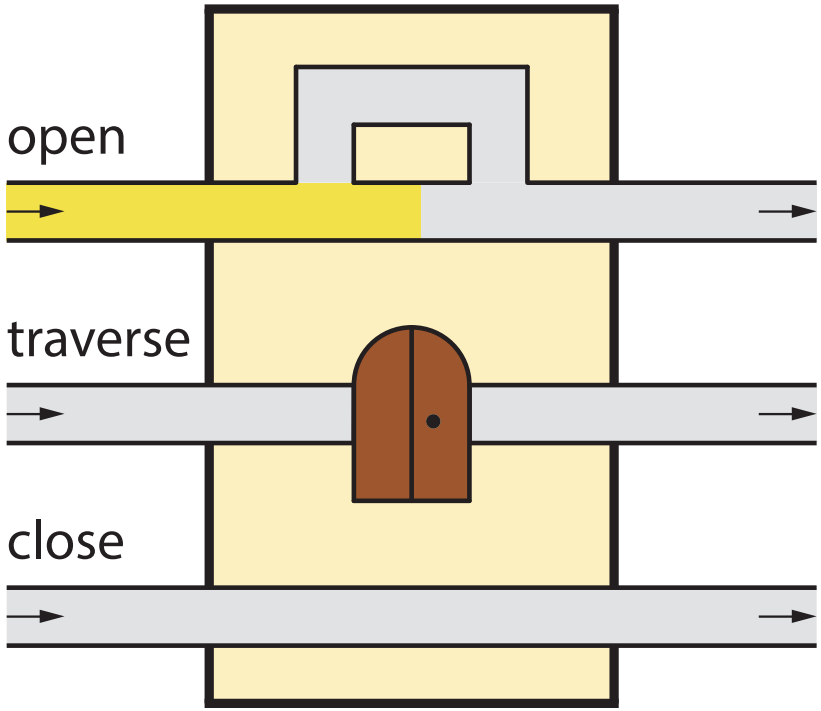

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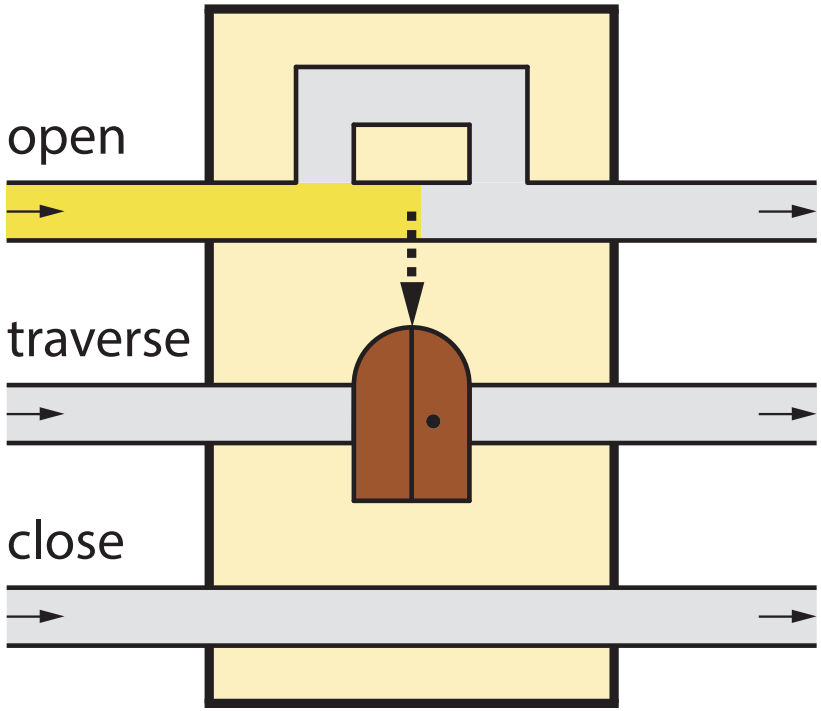

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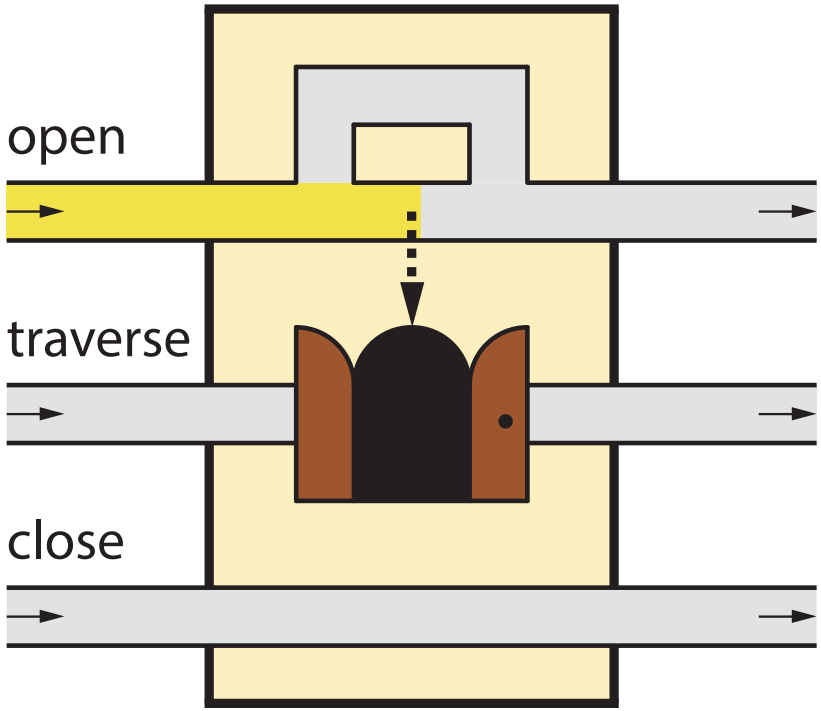

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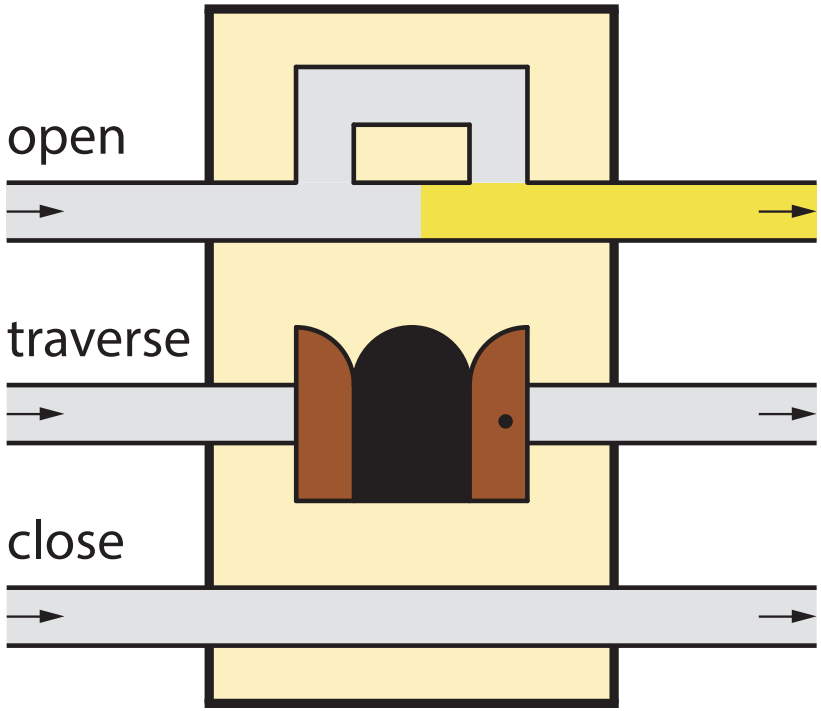
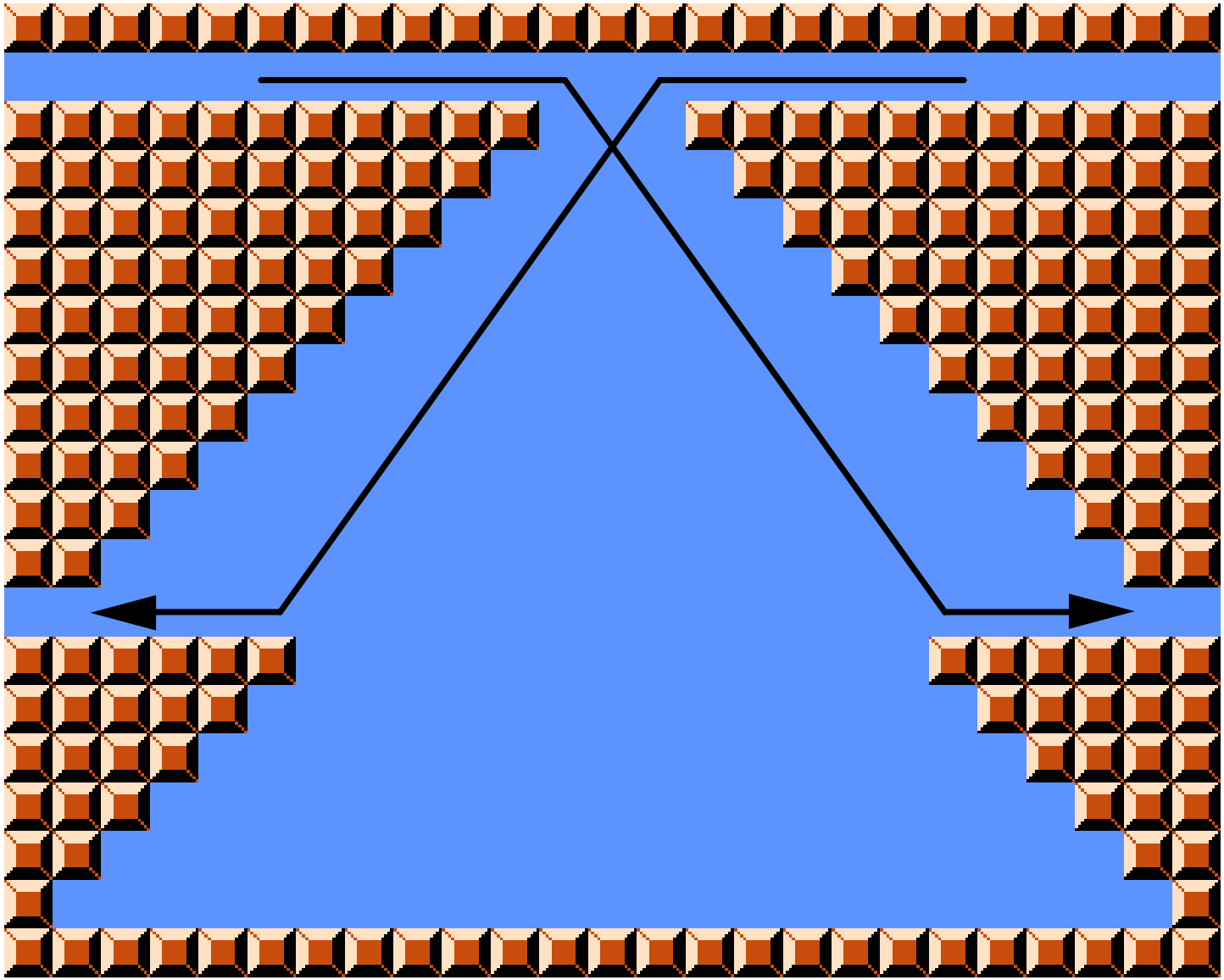
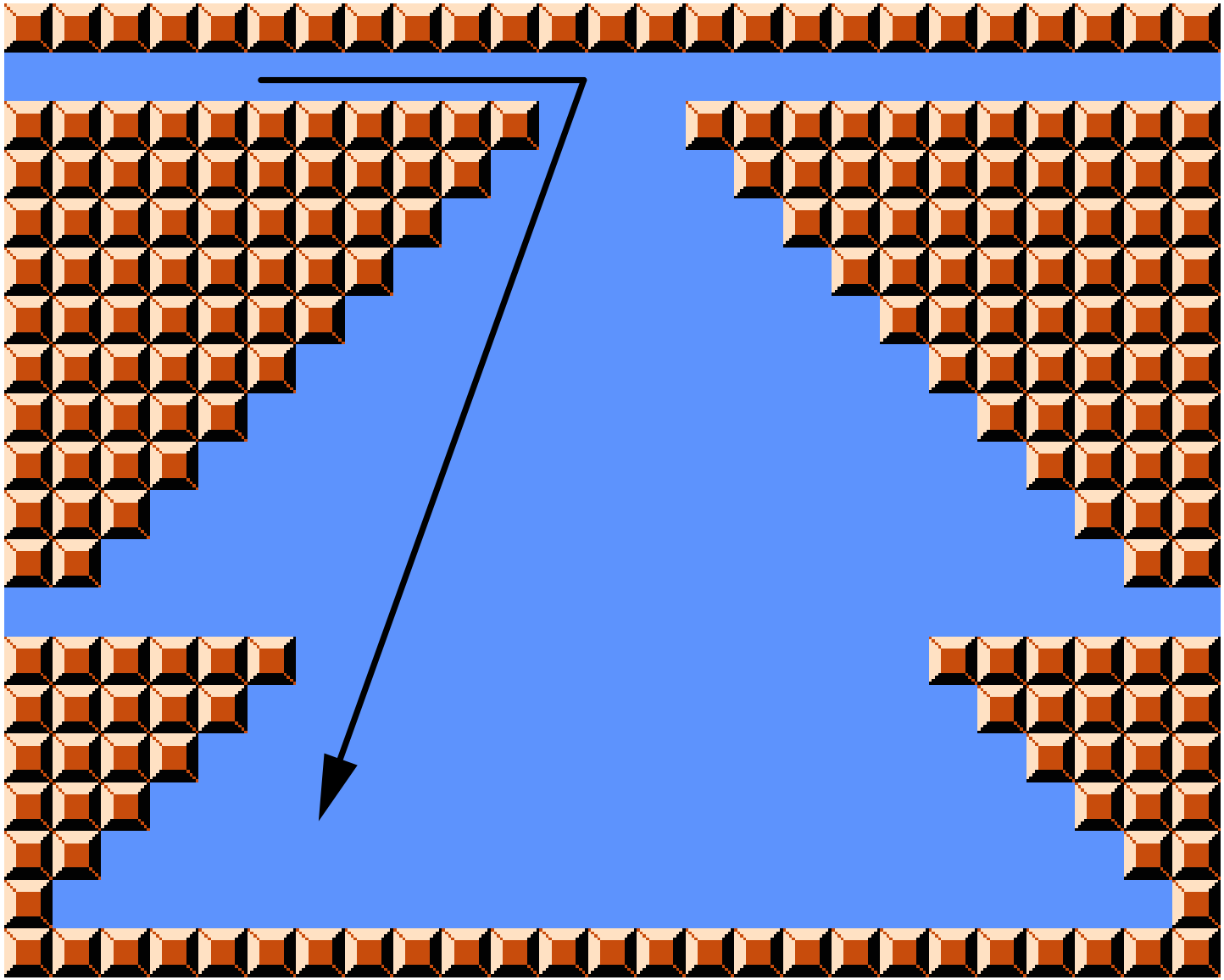


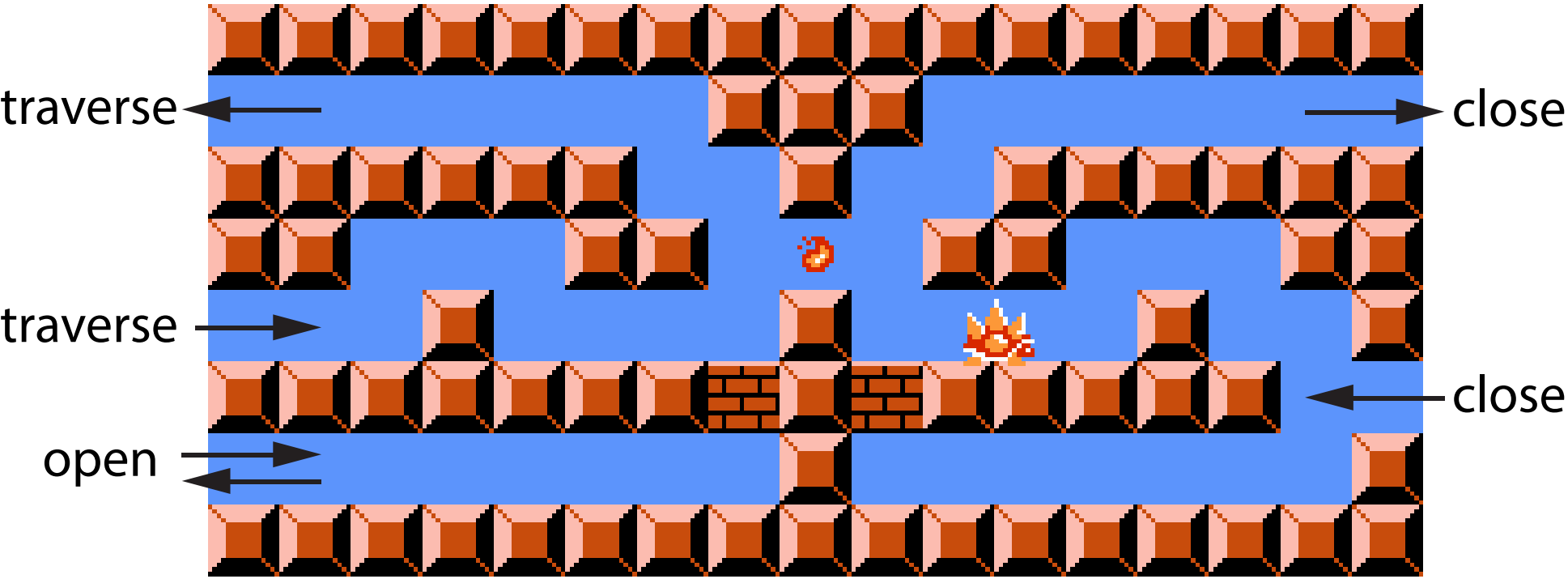
Crossover gadget



Crossover gadget



Door gadget



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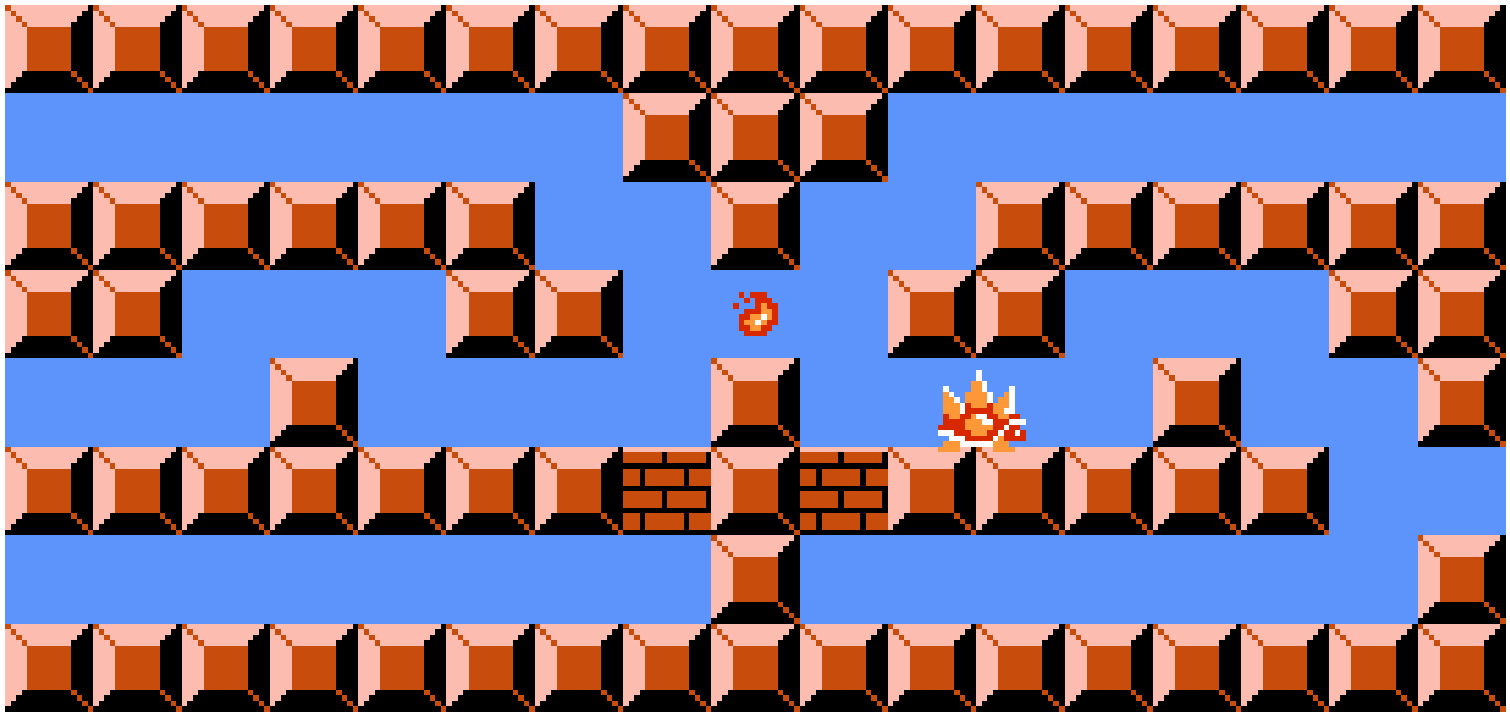

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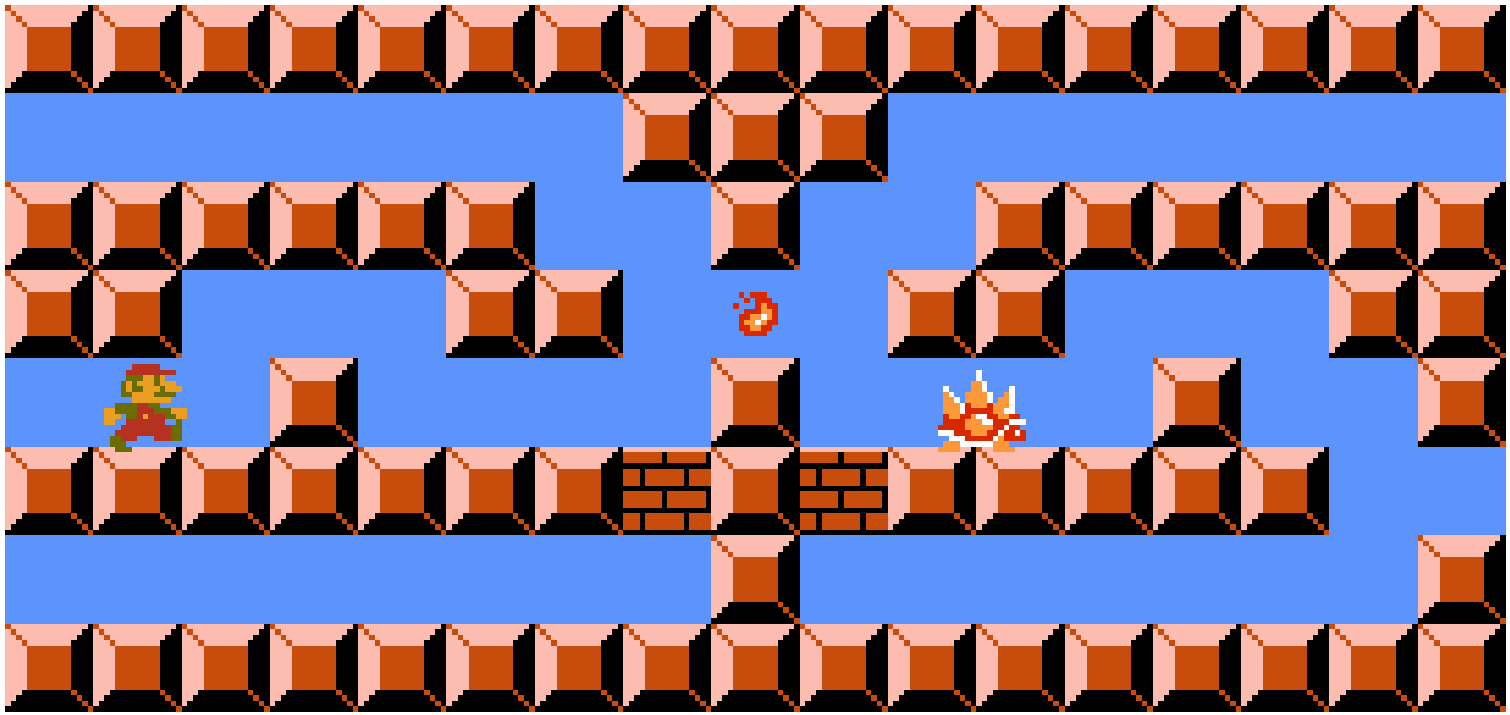

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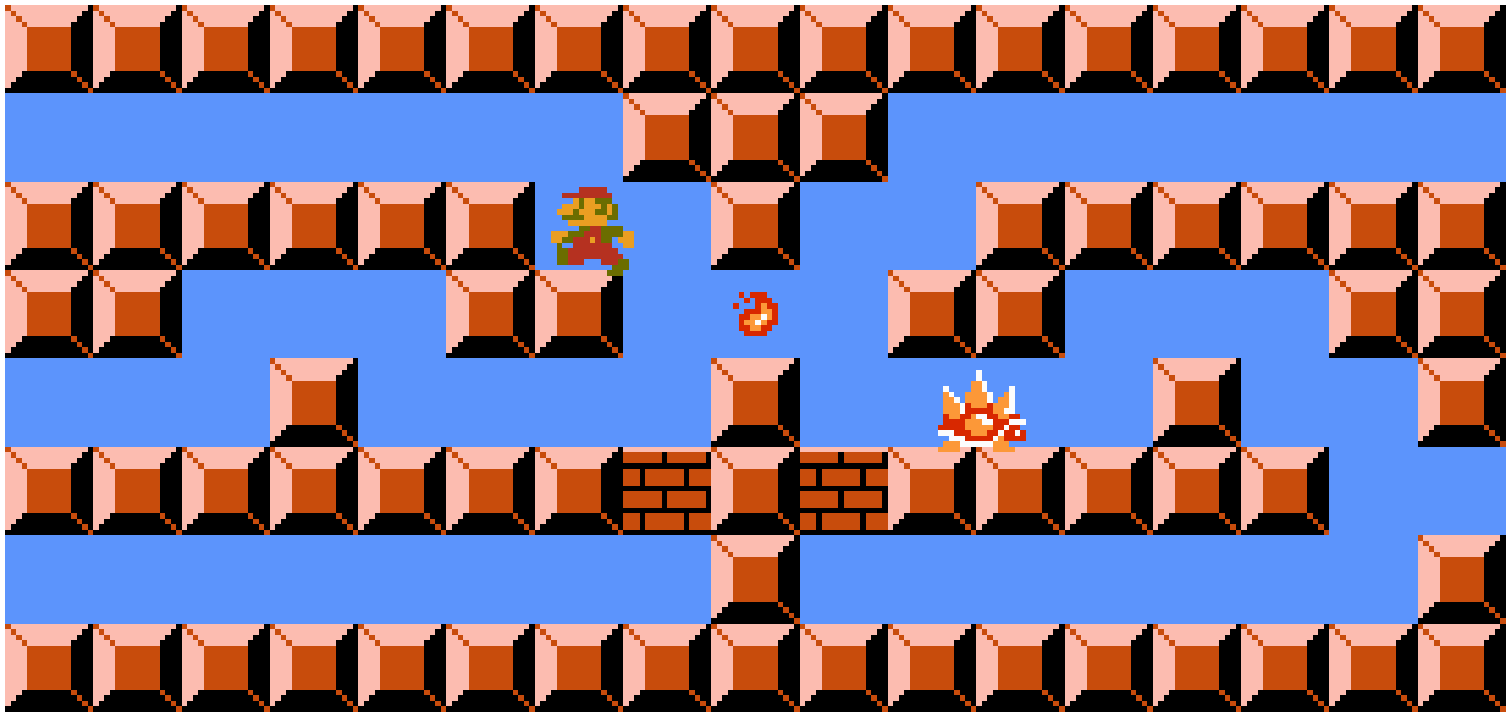

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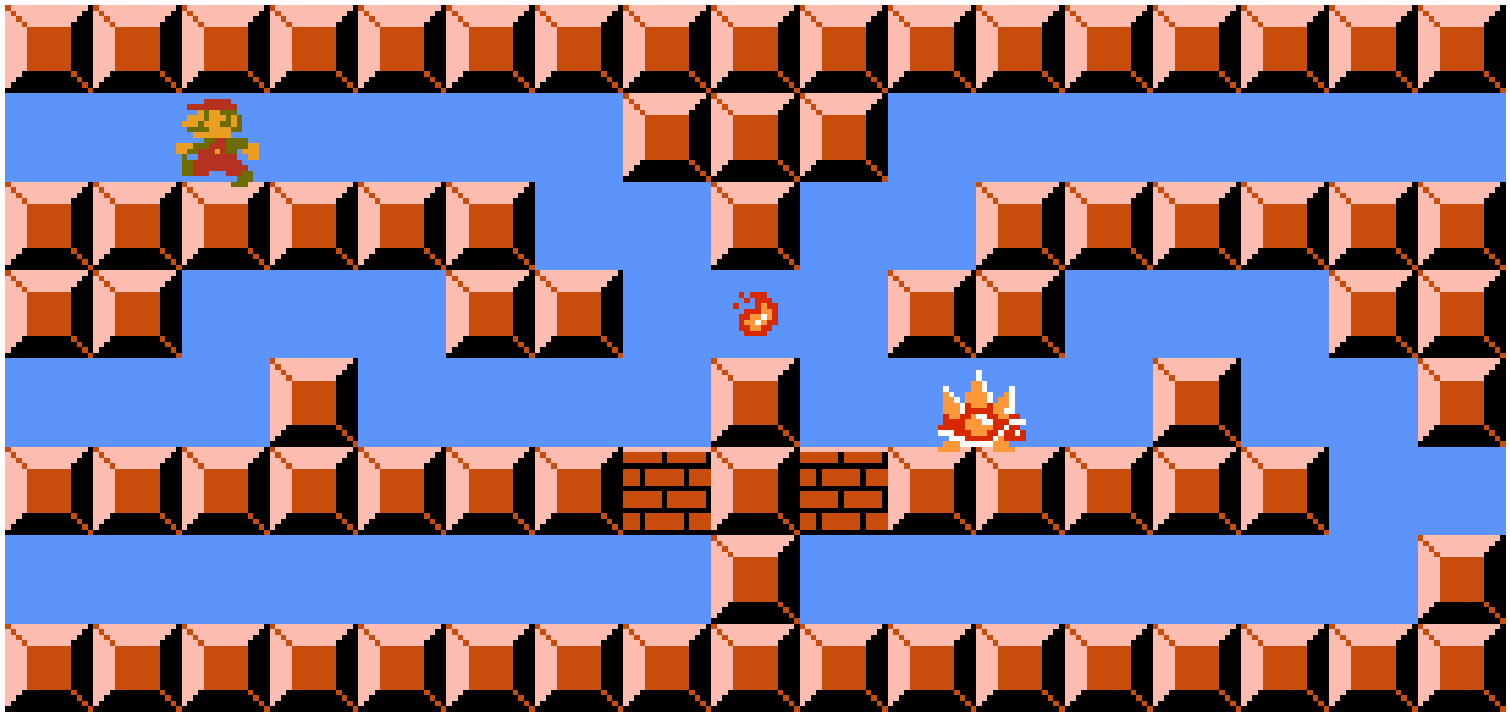

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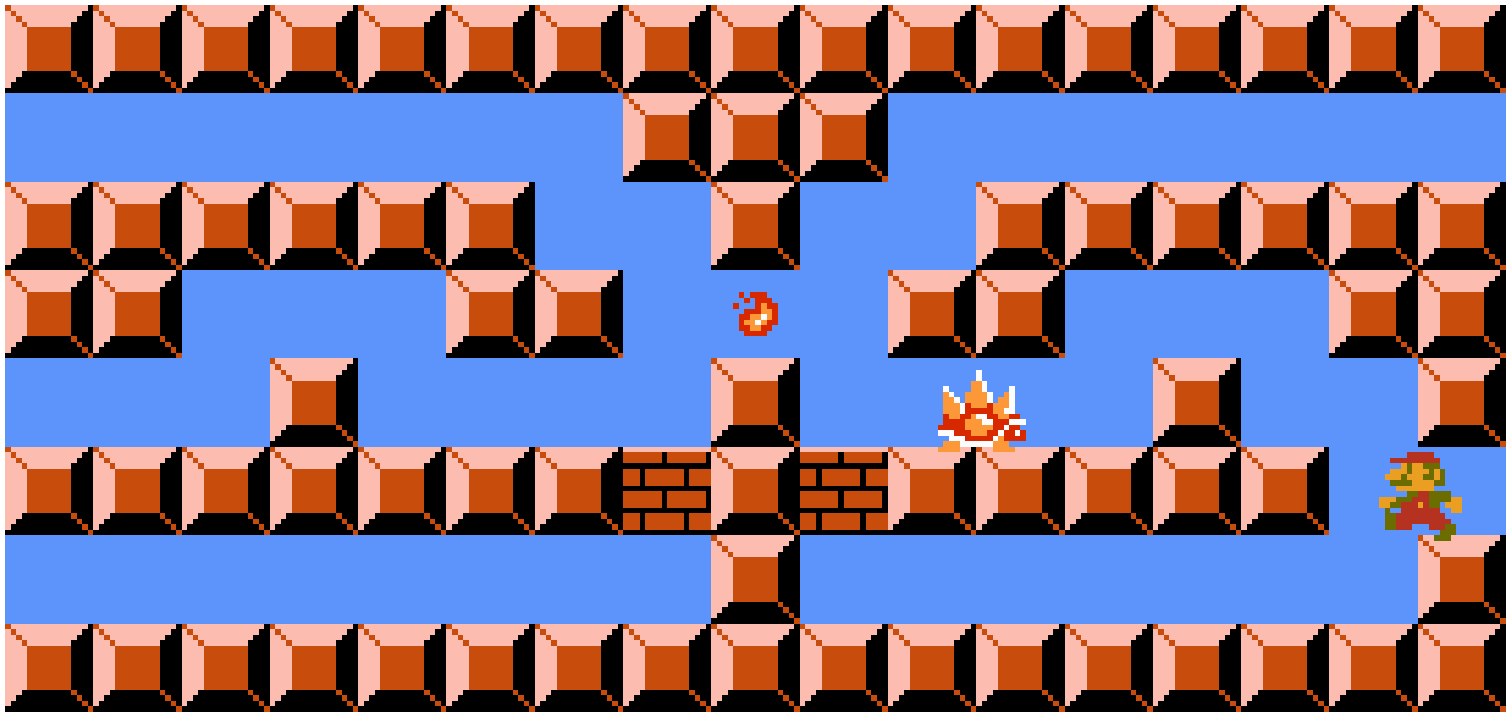

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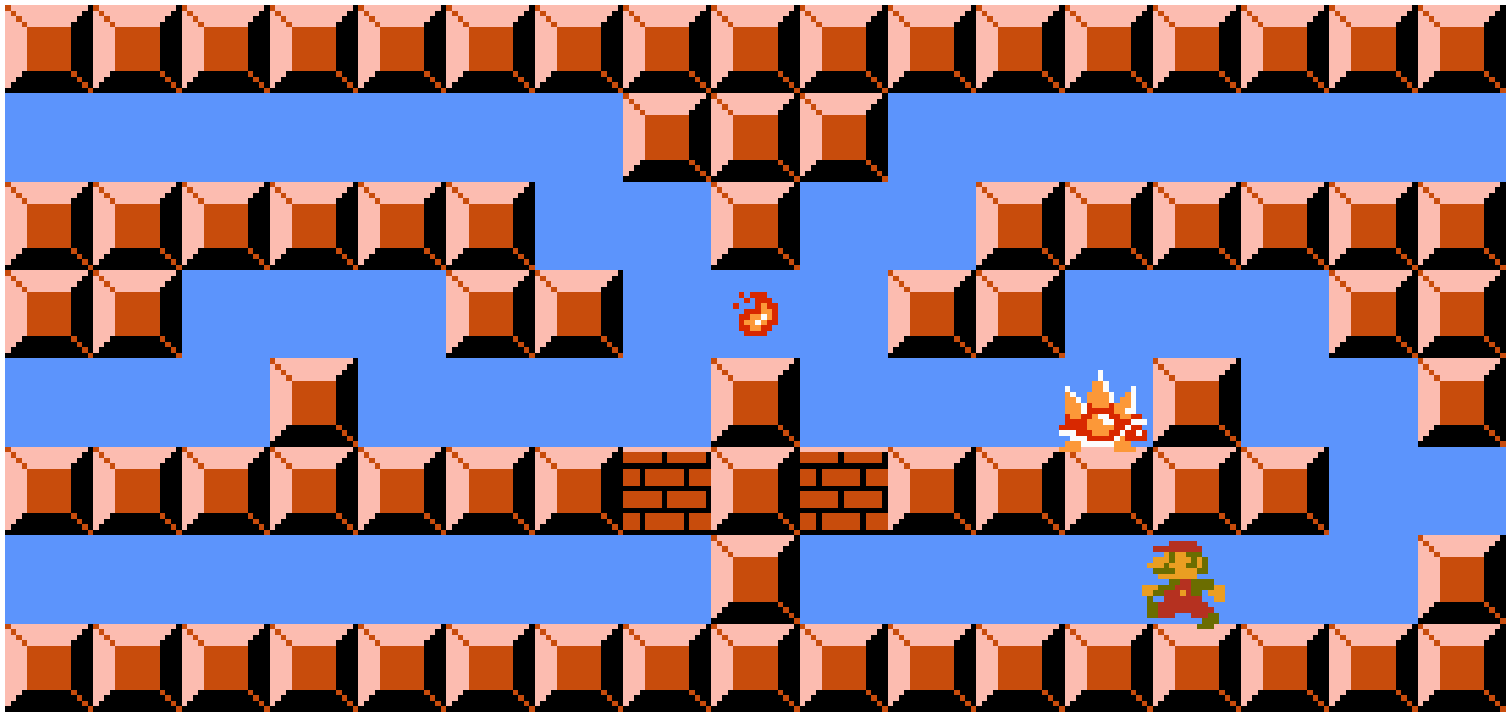

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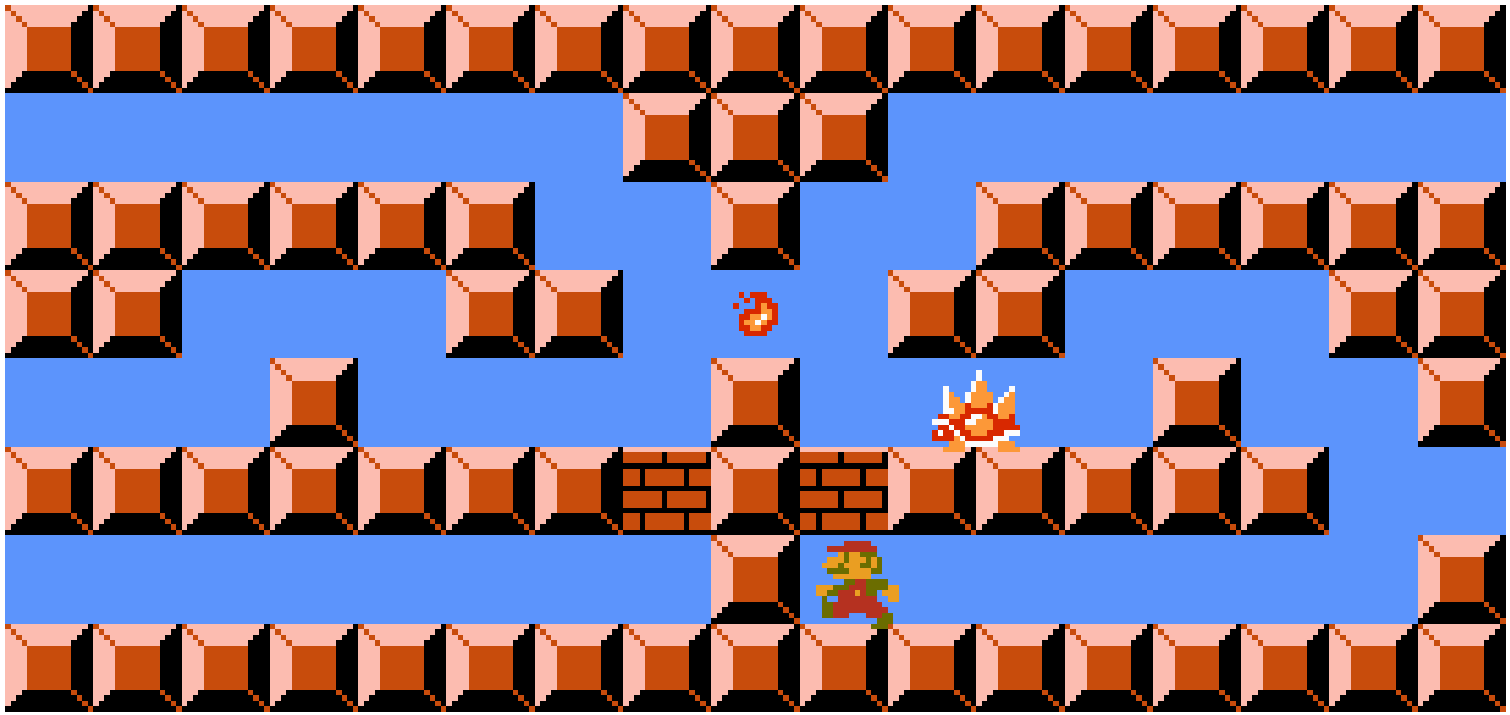

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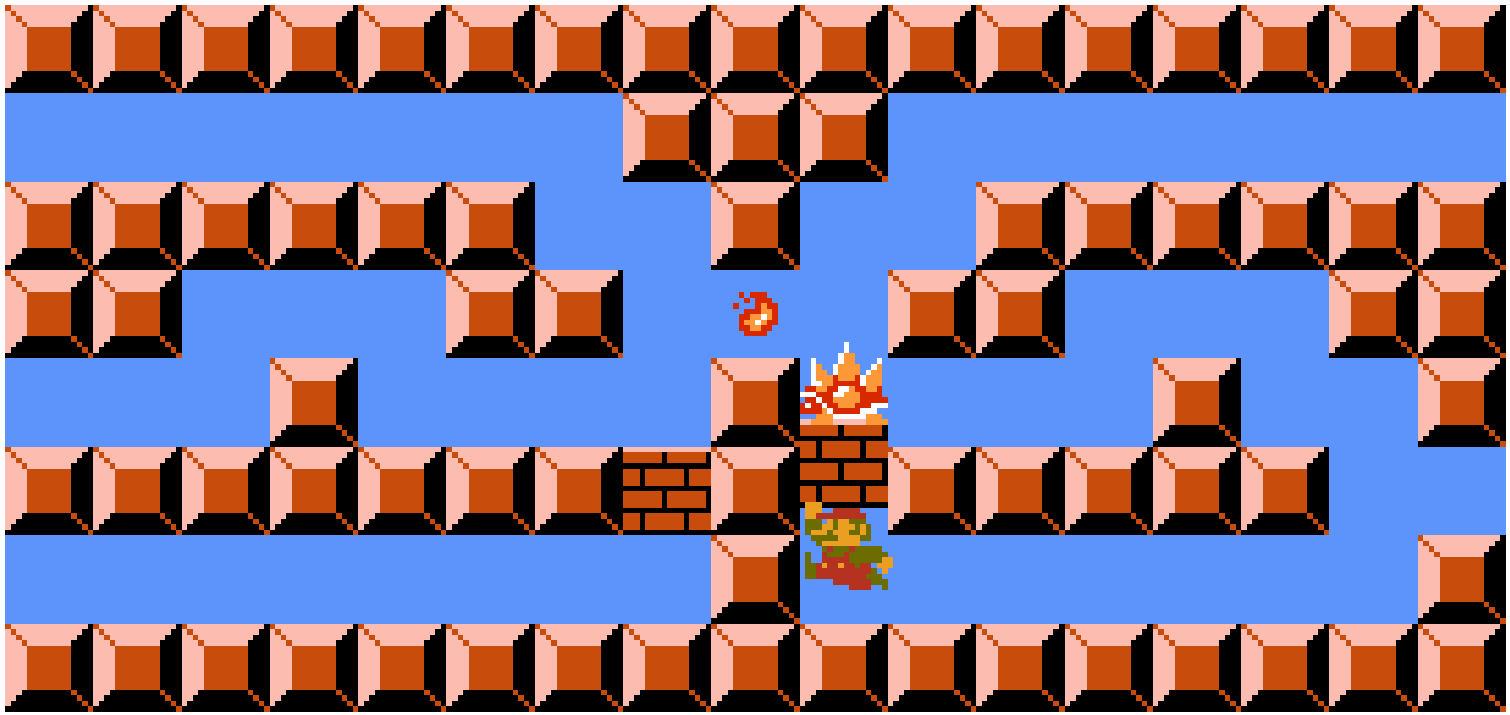

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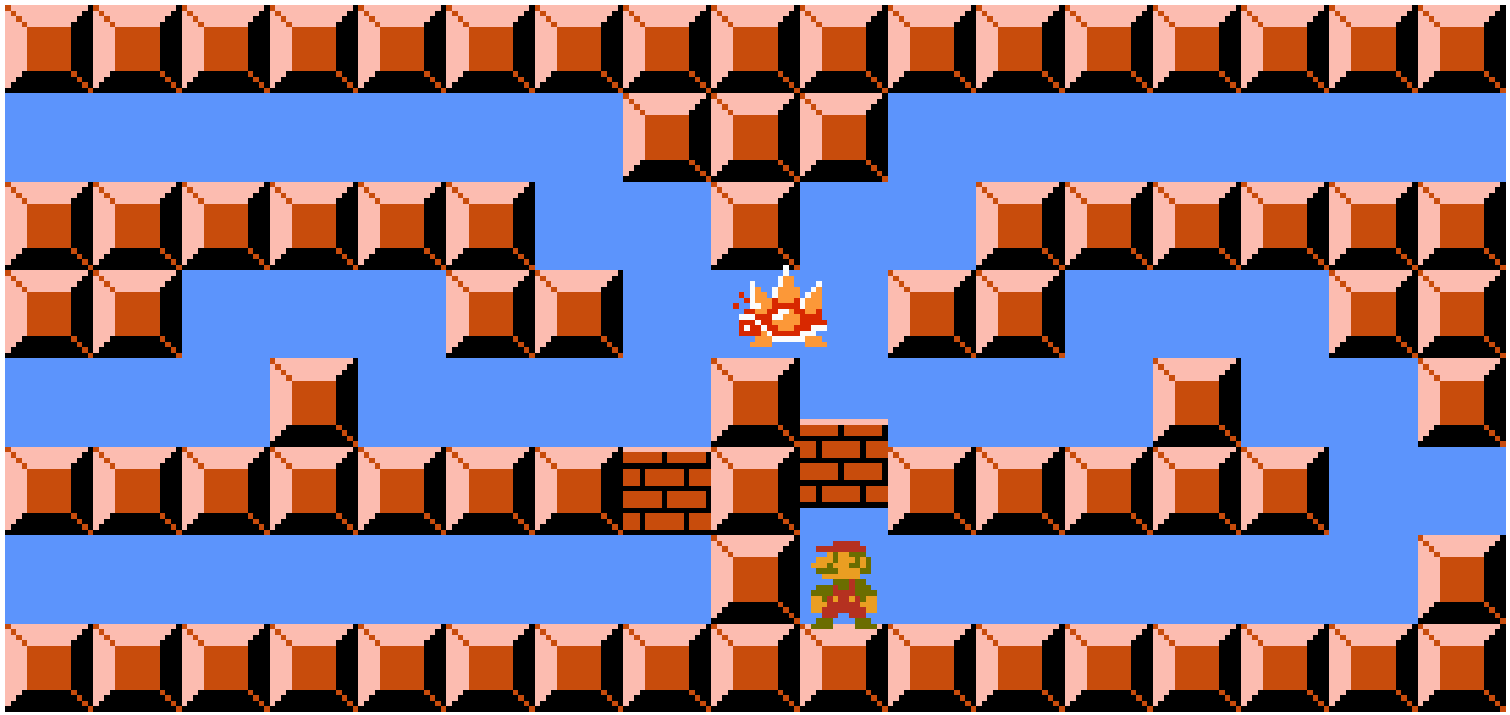

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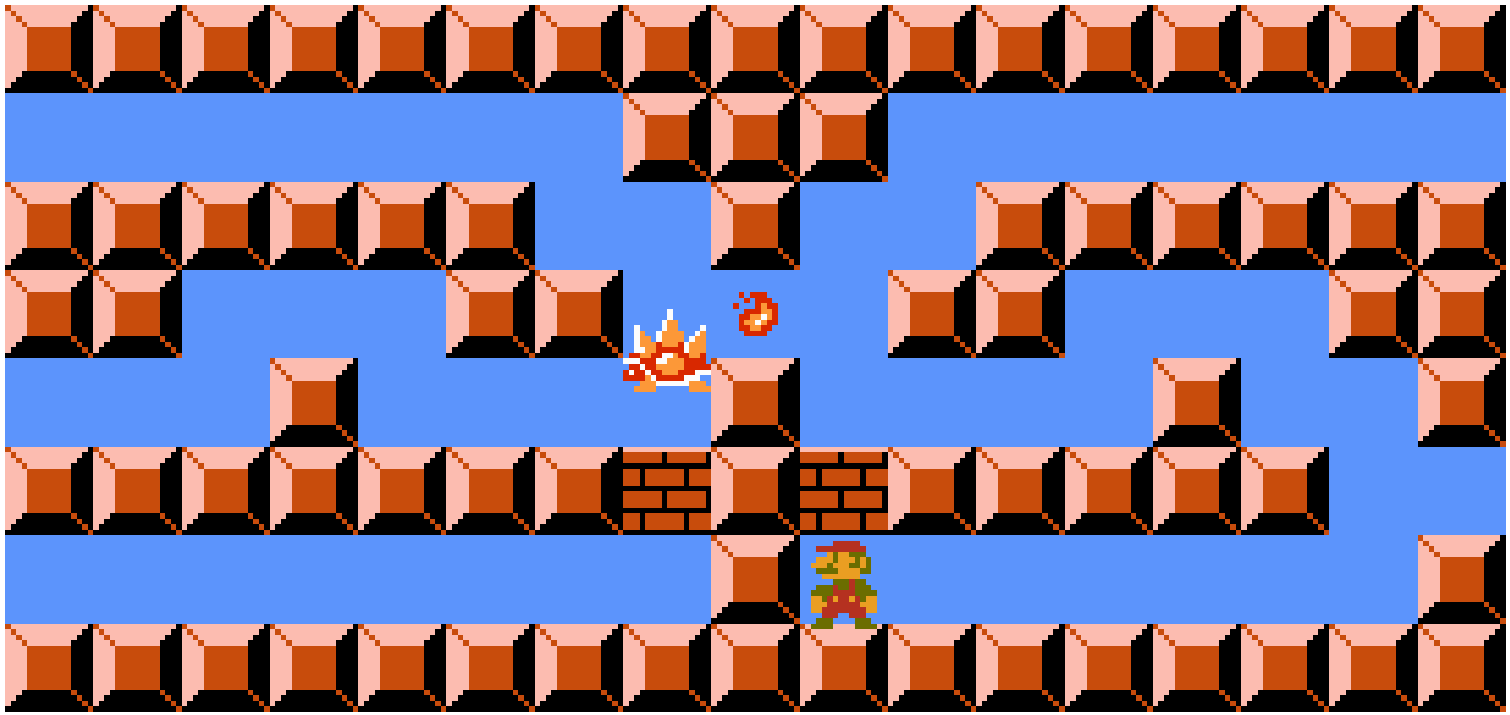

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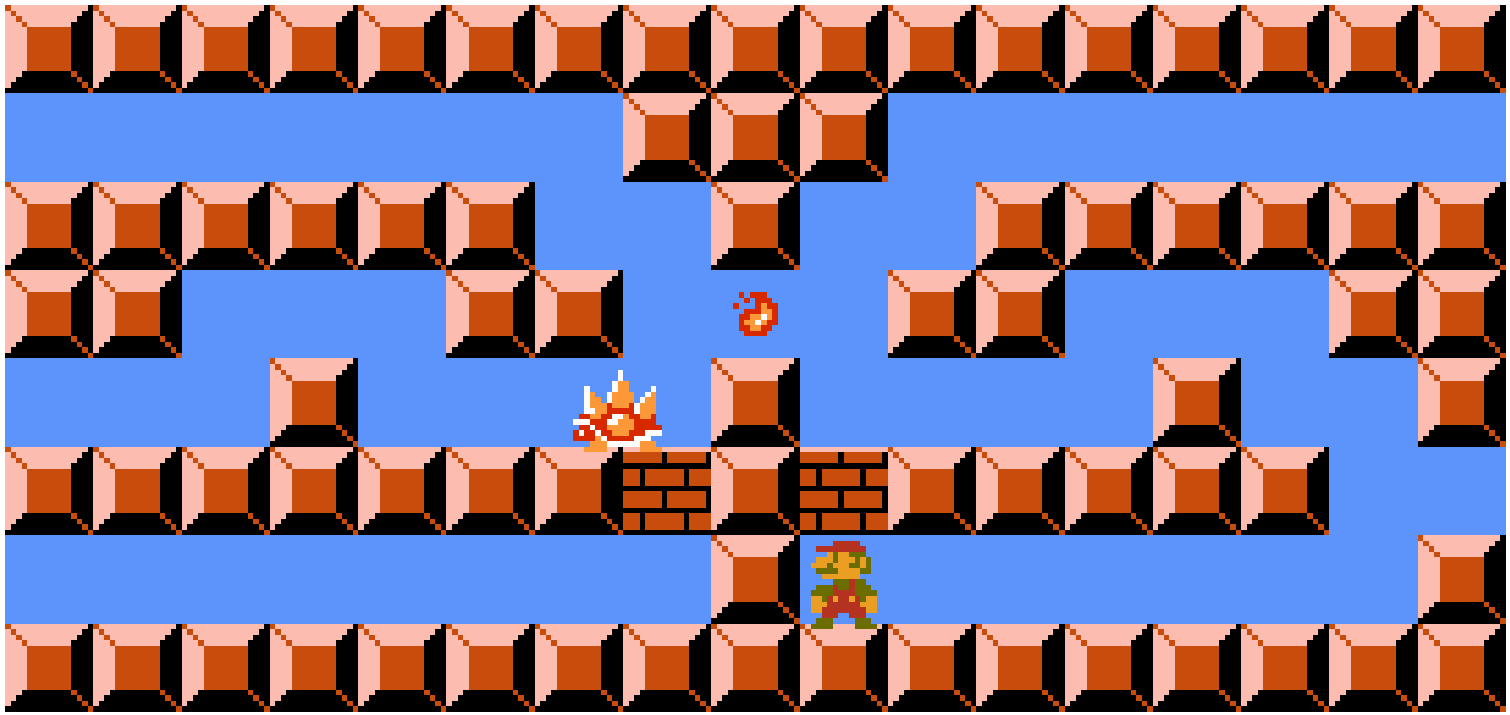

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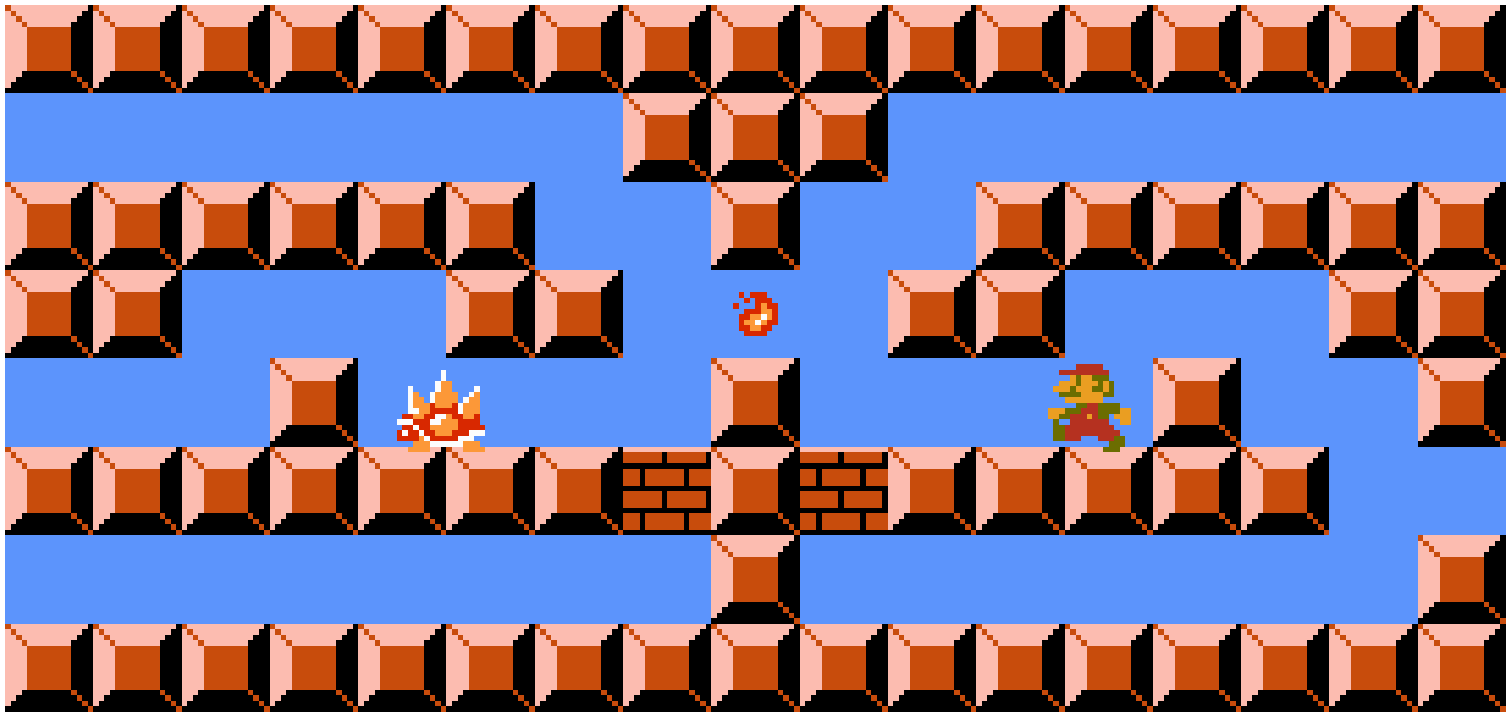

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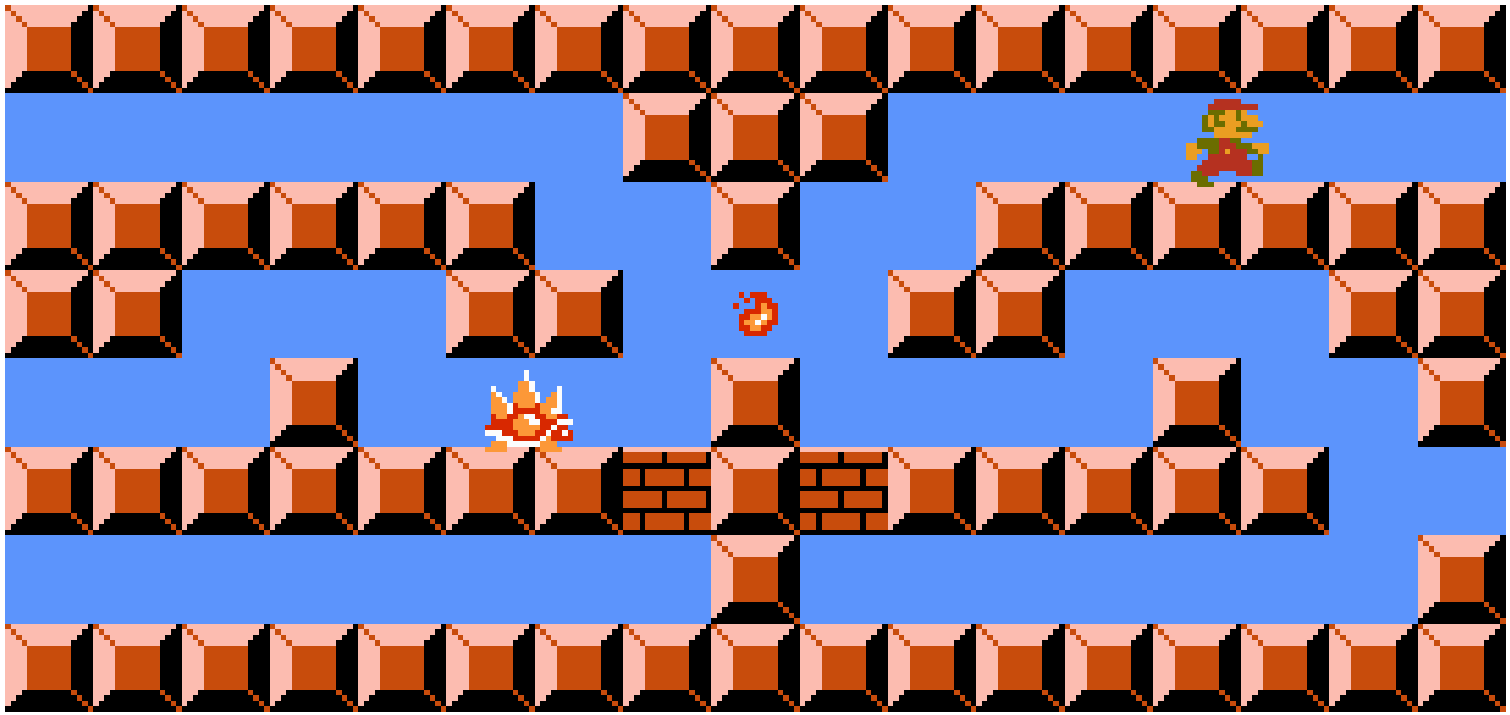

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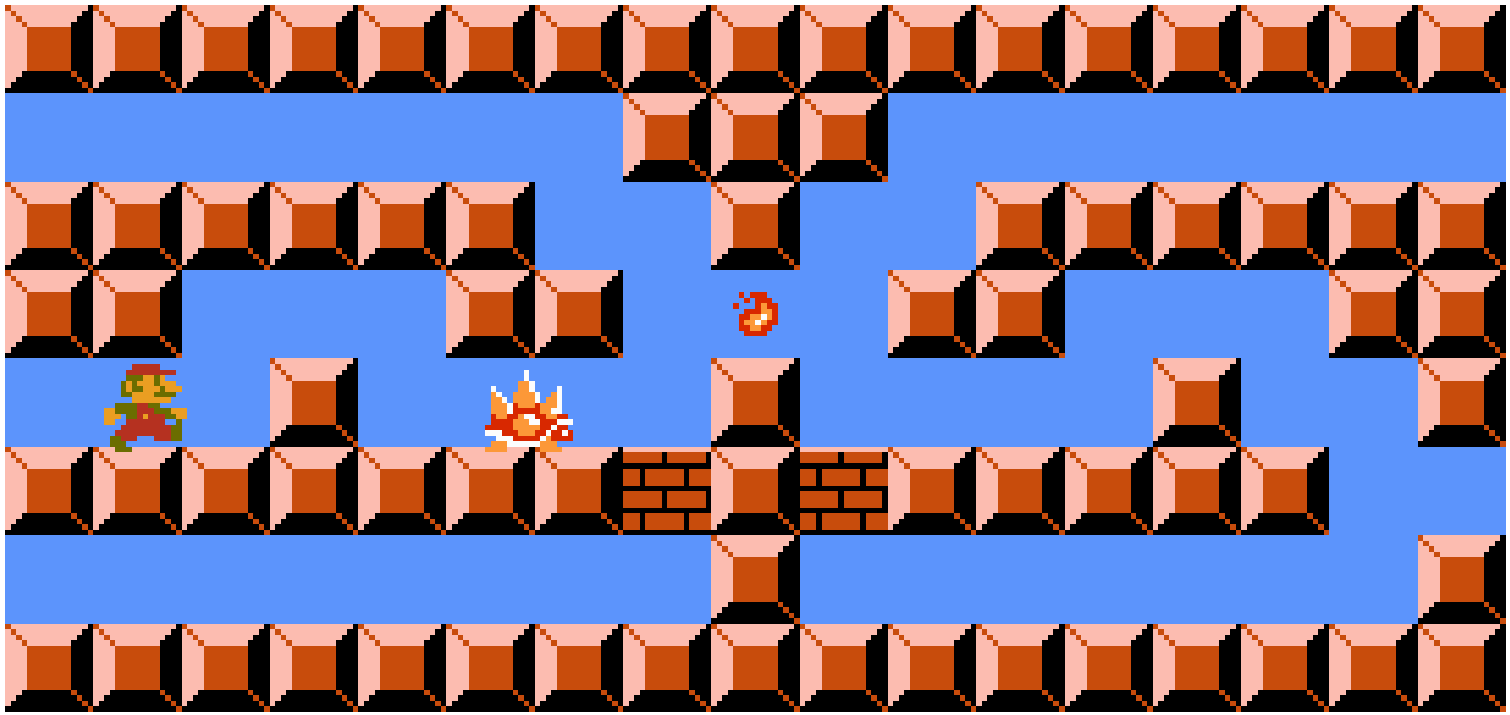
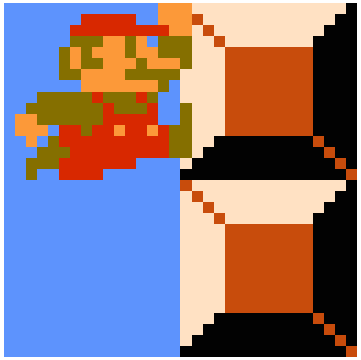
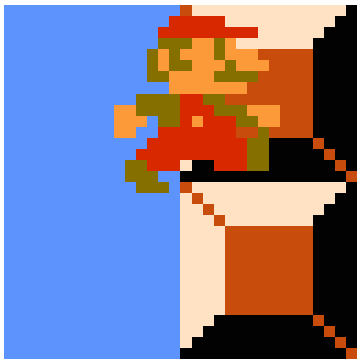


Glitches: walk through walls and walljump



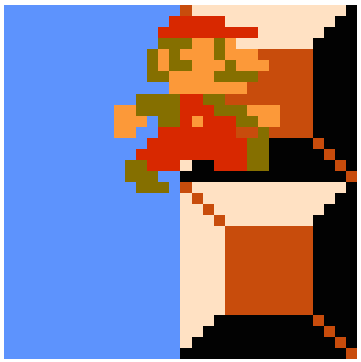
Jump toward a wall and steer in the opposite direction

Glitches: walk through walls and walljump



Jump toward a wall and steer in the opposite direction
If done properly, the wall sucks Mario in instead of pushing him out

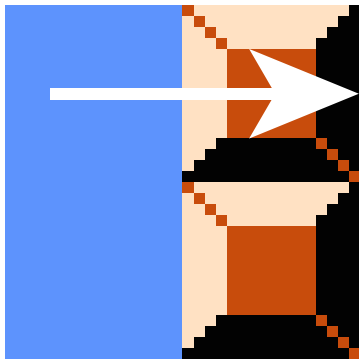
Glitches: walk through walls and walljump

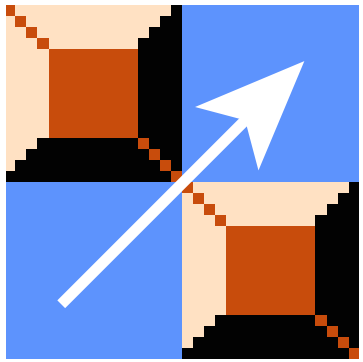


Jump toward a wall and steer in the opposite direction
If done properly, the wall sucks Mario in instead of pushing him out
This allows Mario to walk through certain walls or do a "walljump"

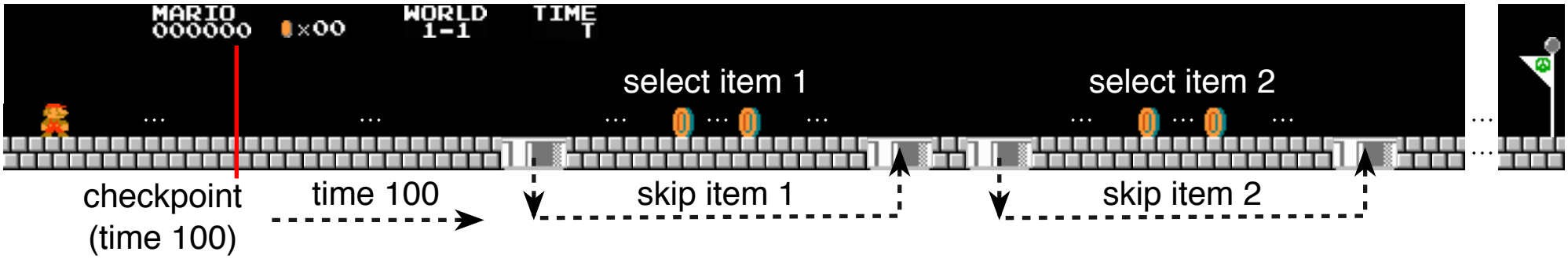
Glitches: walk through walls and walljump

Solution. Avoid the two following configurations:



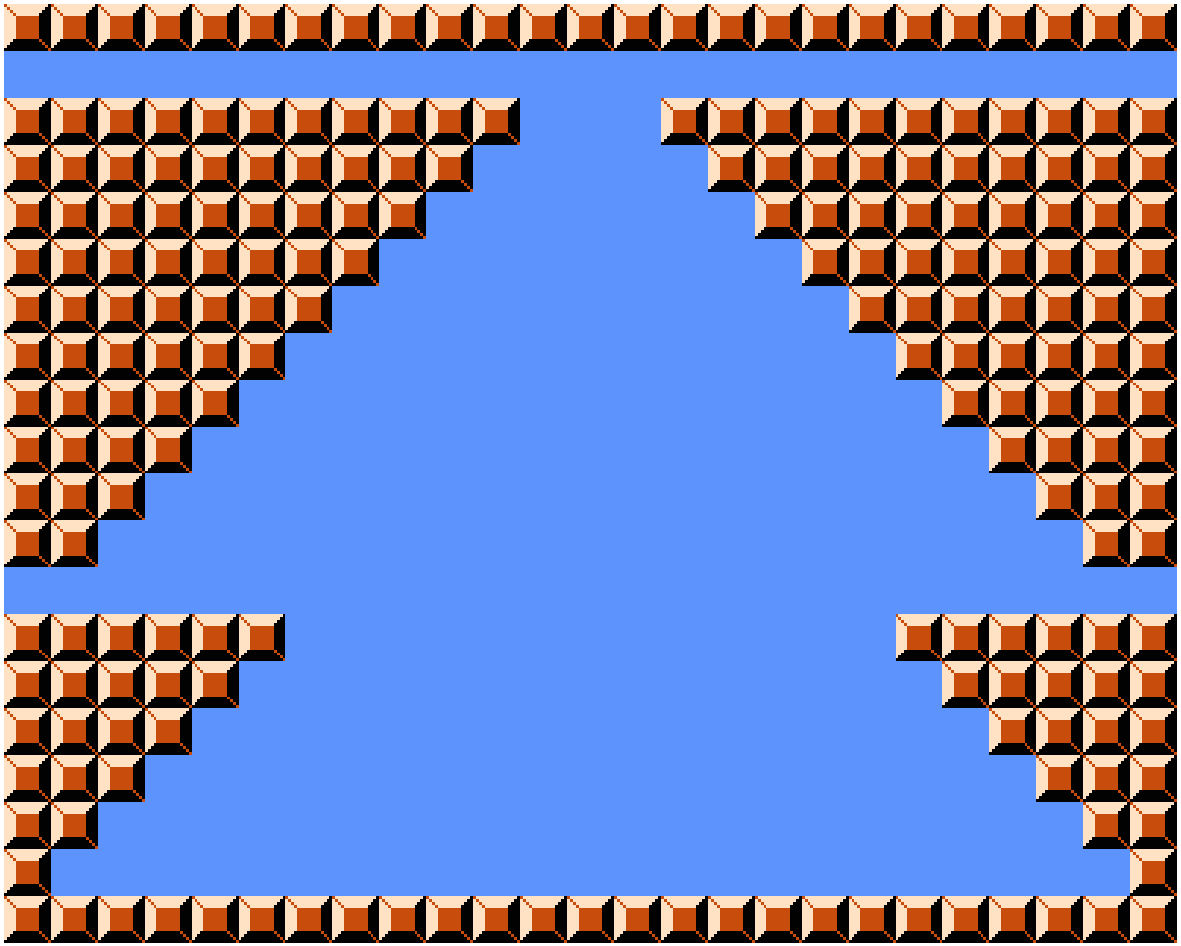


Glitches: walk through walls and walljump



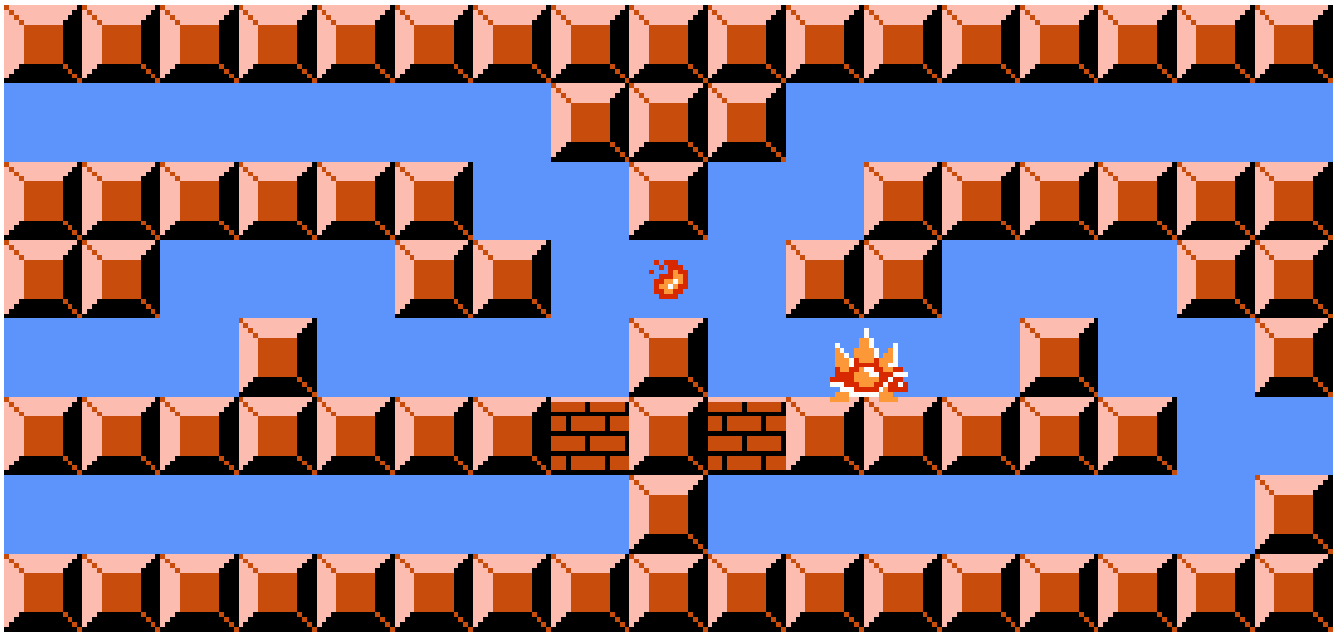
Reduction from Knapsack: OK!

Glitches: walk through walls and walljump



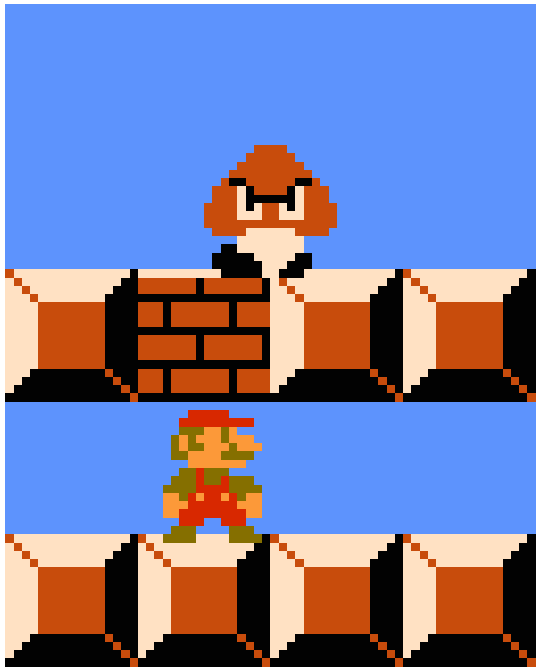
Crossover gadget: OK!

Glitches: walk through walls and walljump



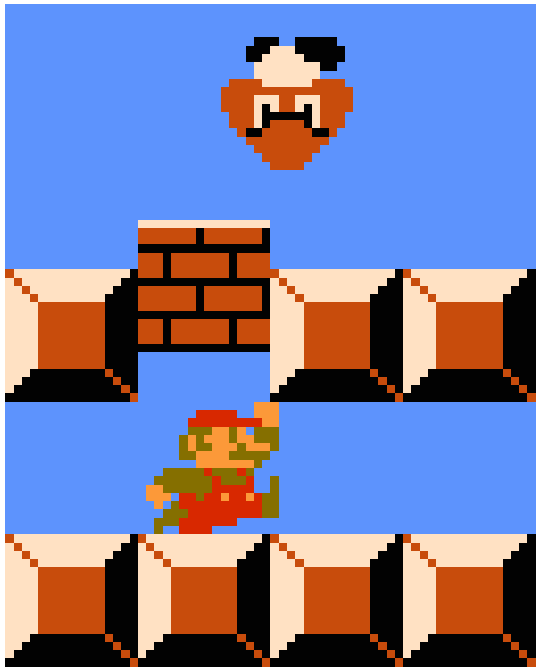
Door gadget: OK!

Glitches: walk through walls and walljump



If there is a monster on top of a brick block, Mario can jump through it by quickly hitting it twice from below

Glitches: walk through walls and walljump



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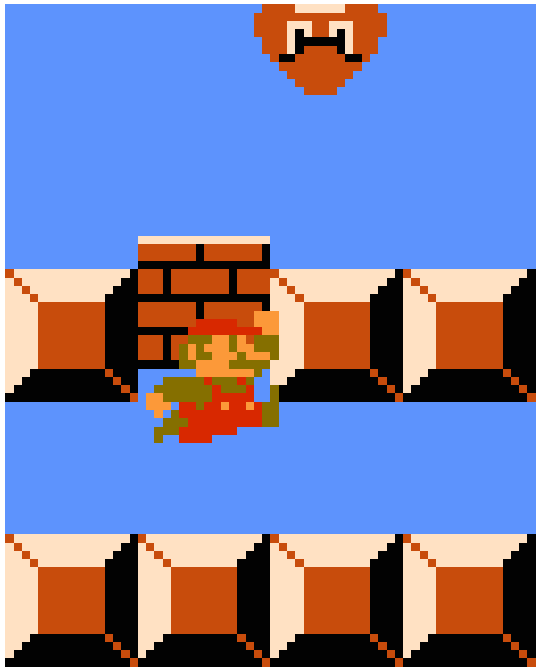
Glitches: walk through walls and walljump



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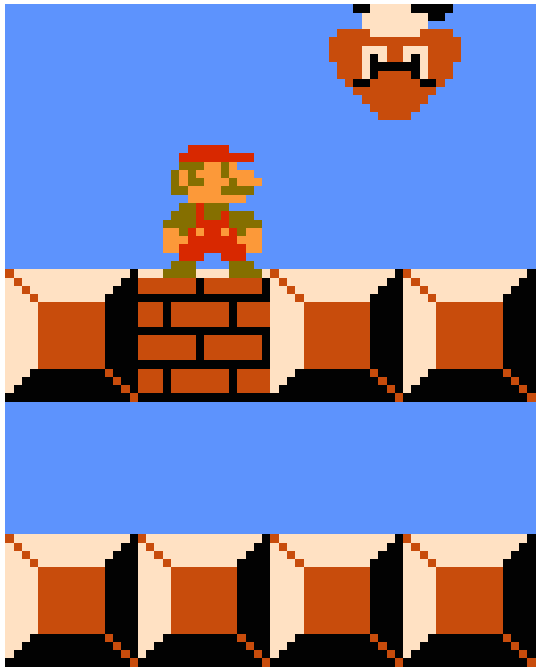
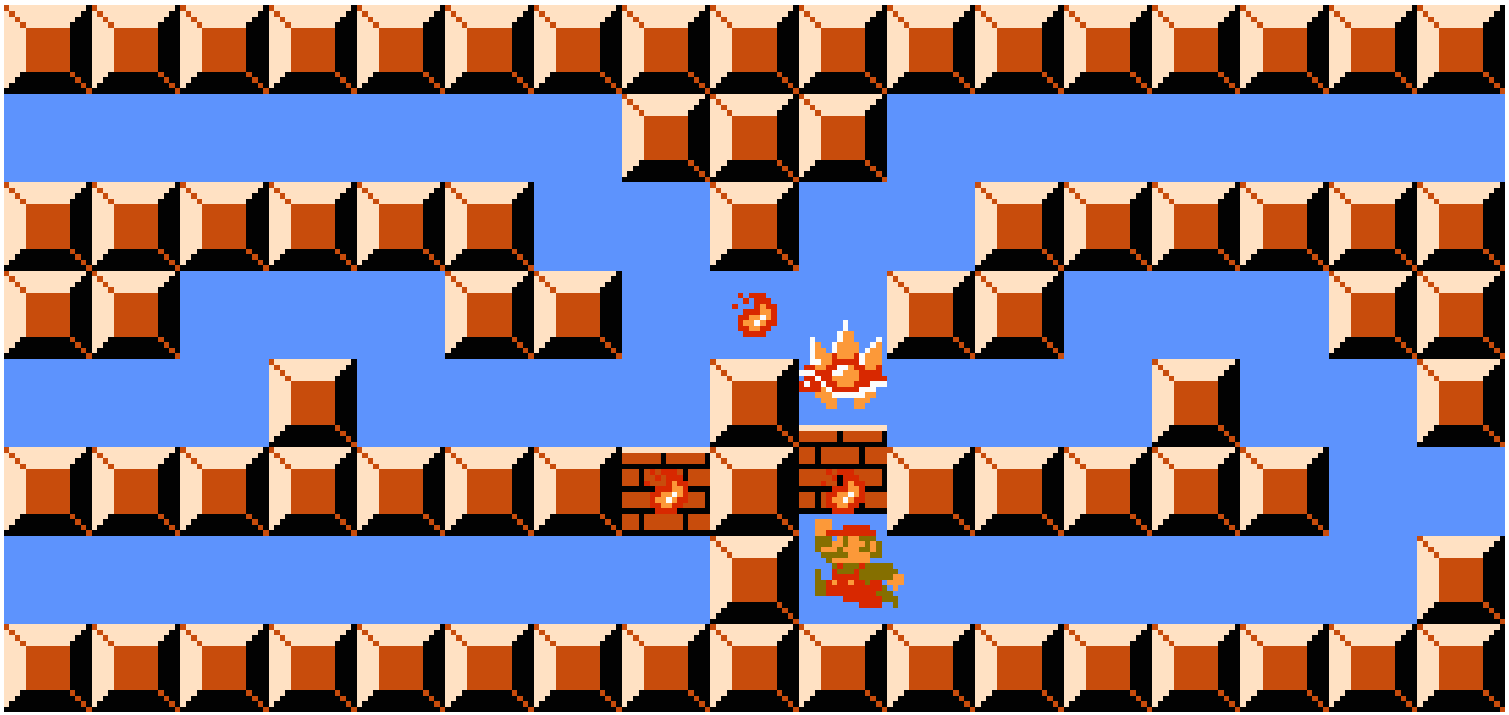
Glitches: walk through walls and walljump



If there is a monster on top of a brick block, Mario can jump through it by quickly hitting it twice from below

Glitches: walk through walls and walljump

Solution. Attach firebars to brick blocks

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WORLD
8-4
TIME
171
THANK YOU MARIO!
YOUR QUEST IS OVER.