Getting Close Without Touching

SIROCCO 2012

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CORDA robot model

Robots in CORDA model have:

- Motorial capabilities
  (freely move in a 2-dimensional plane)
- Sensorial capabilities
  (sense the positions of other robots in the plane)
- No explicit way of communicating

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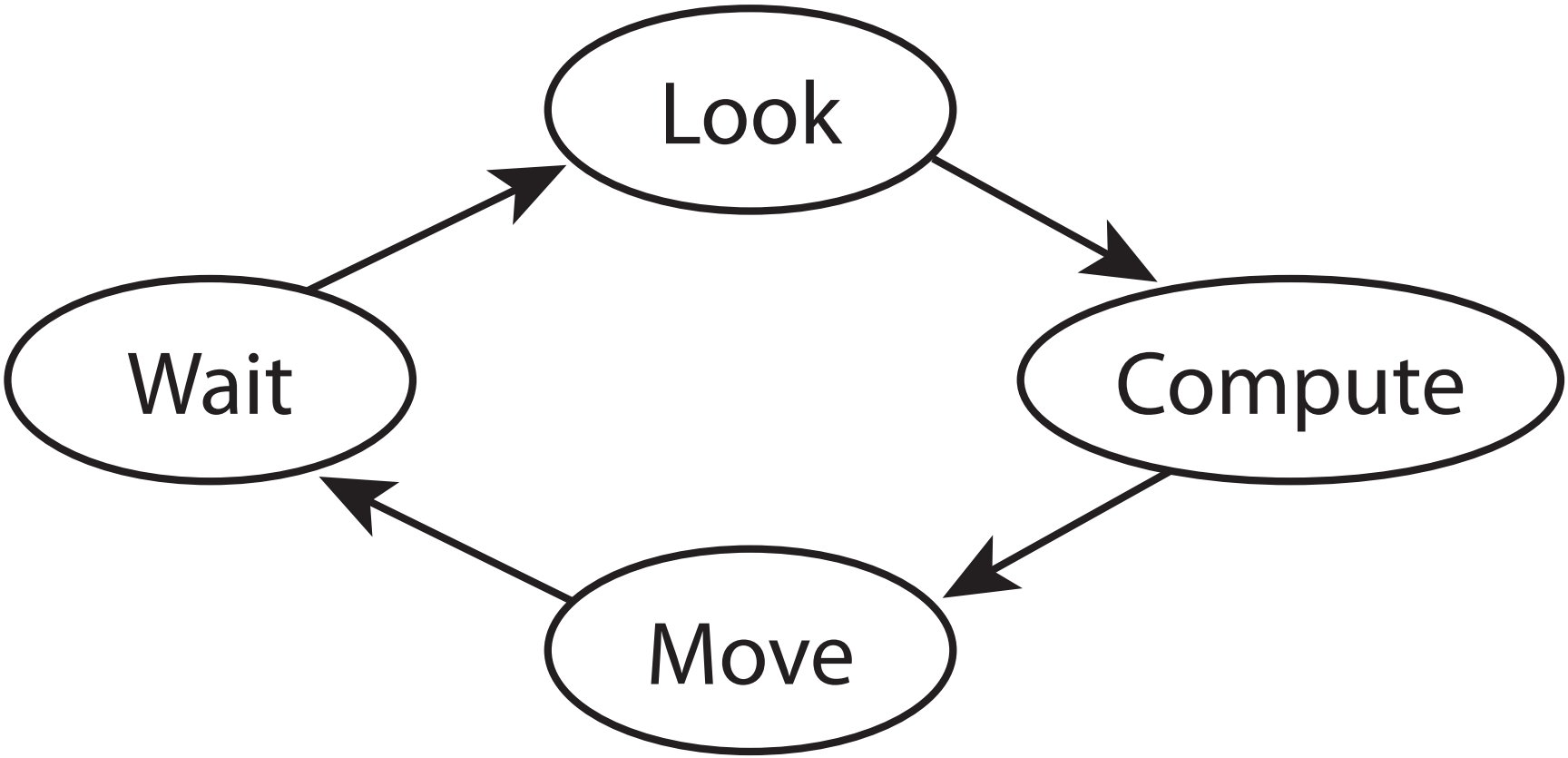
Additionally, they are:

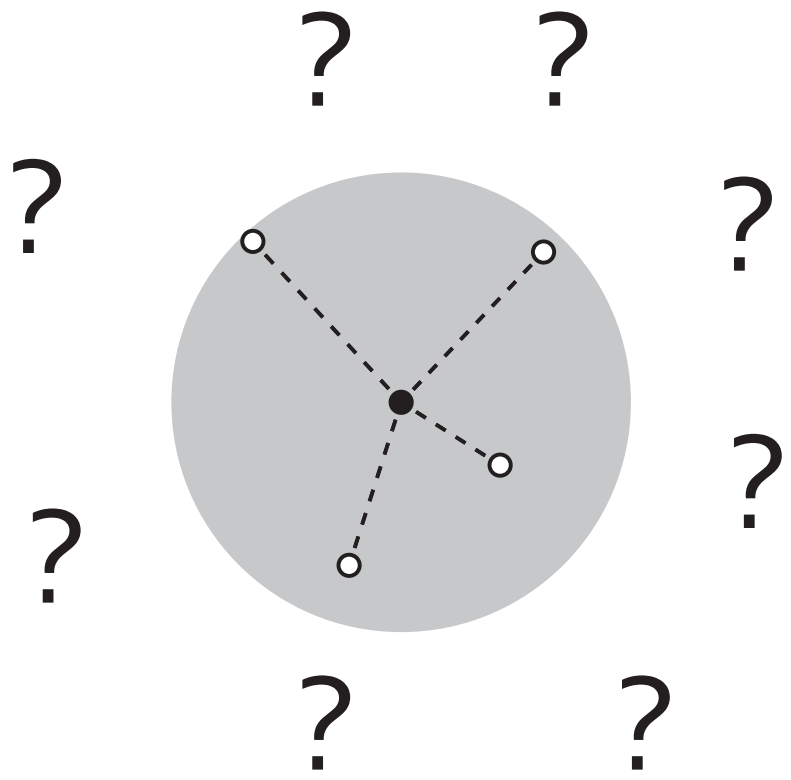
- Homogeneous (all executing the same algorithm)
- Autonomous (no centralized control)
- Oblivious (no memory of past events)
- Anonymous (no unique identifiers)
- Asynchronous (no global timer)

Robot life cycle



Limited visibility

We want robots to sense each other only if they are close enough.



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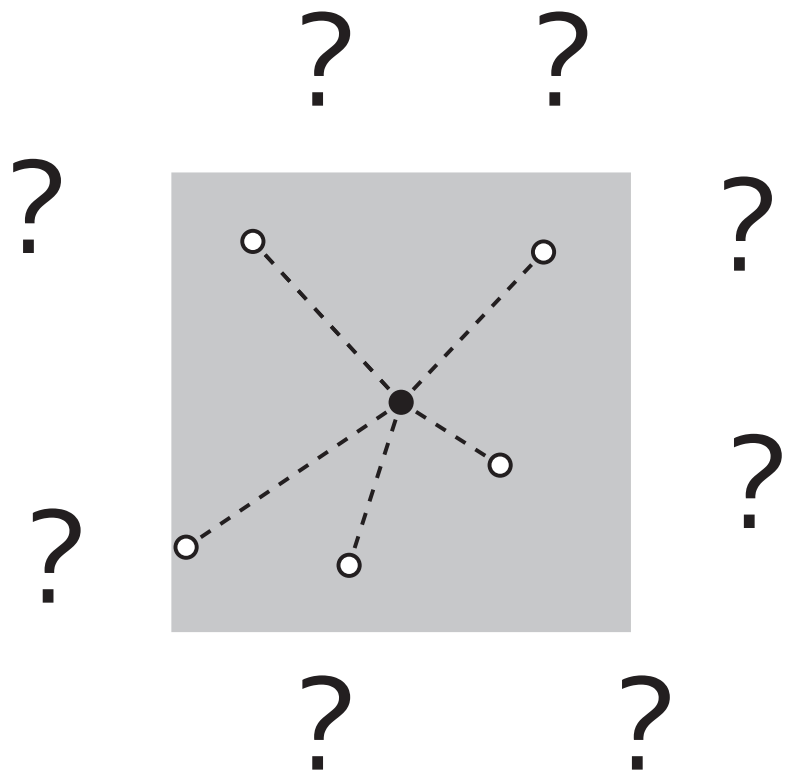
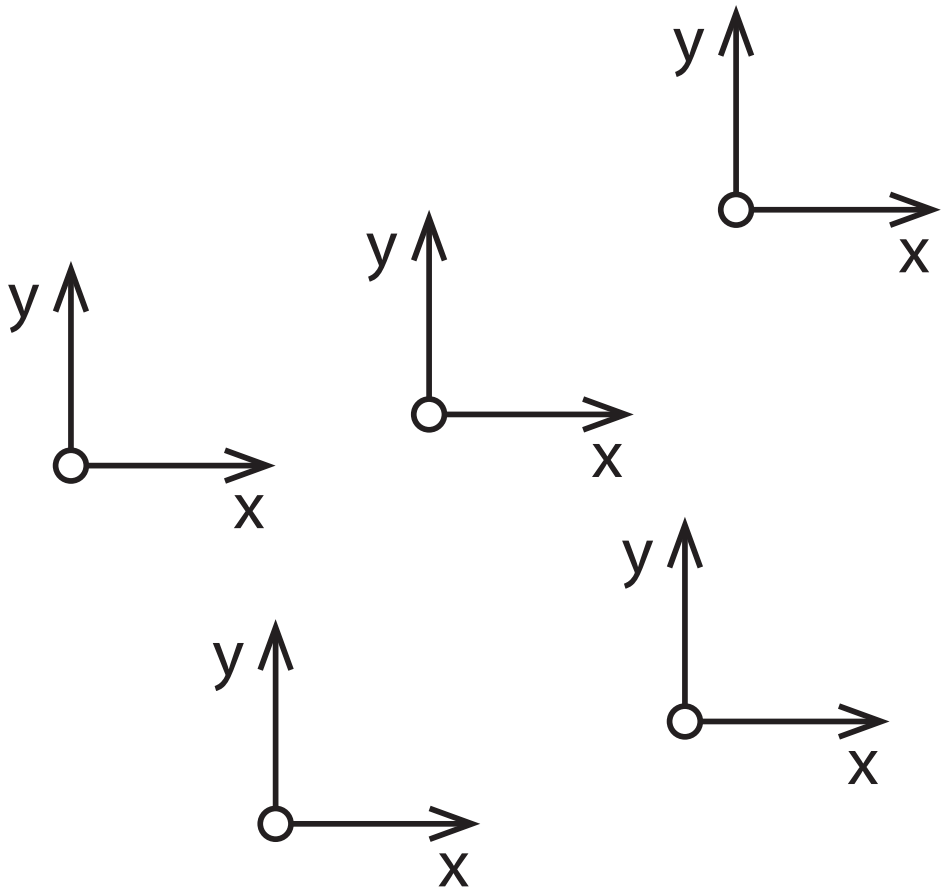


Actually, the distance function we consider is the one induced by the infinity norm.

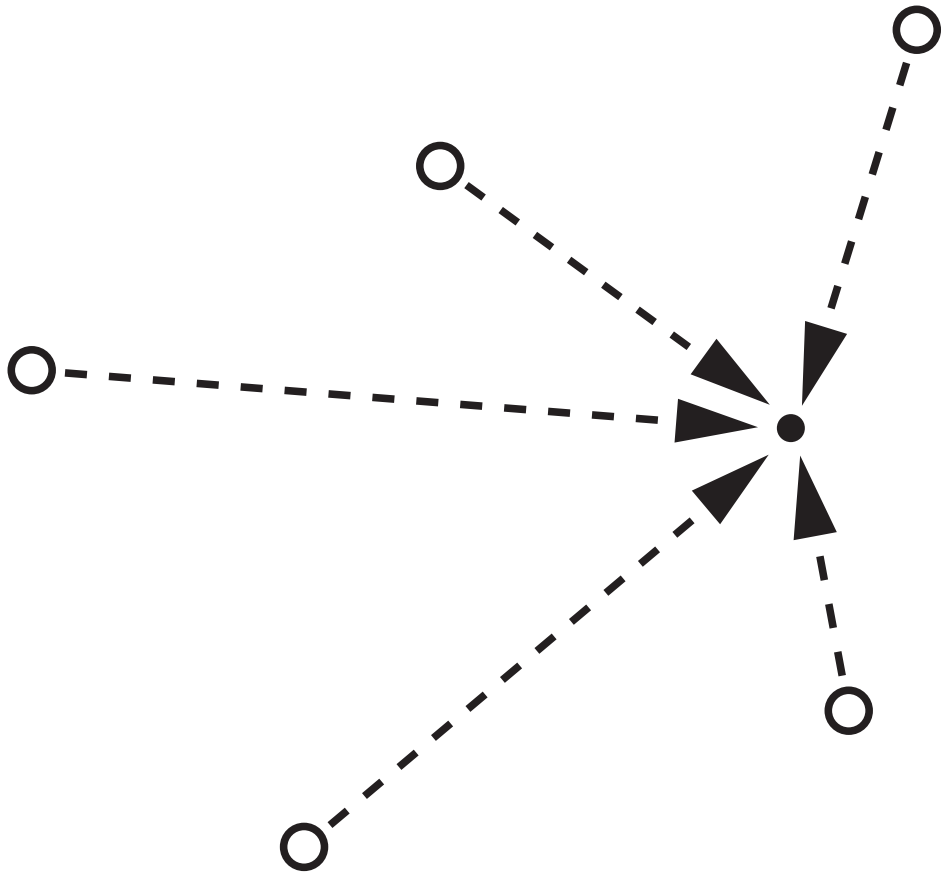
Coordinate system agreement

Each robot has its own coordinate system, but they all agree on axis directions and unit of length.



Gathering problem

All robots must gather in a point.



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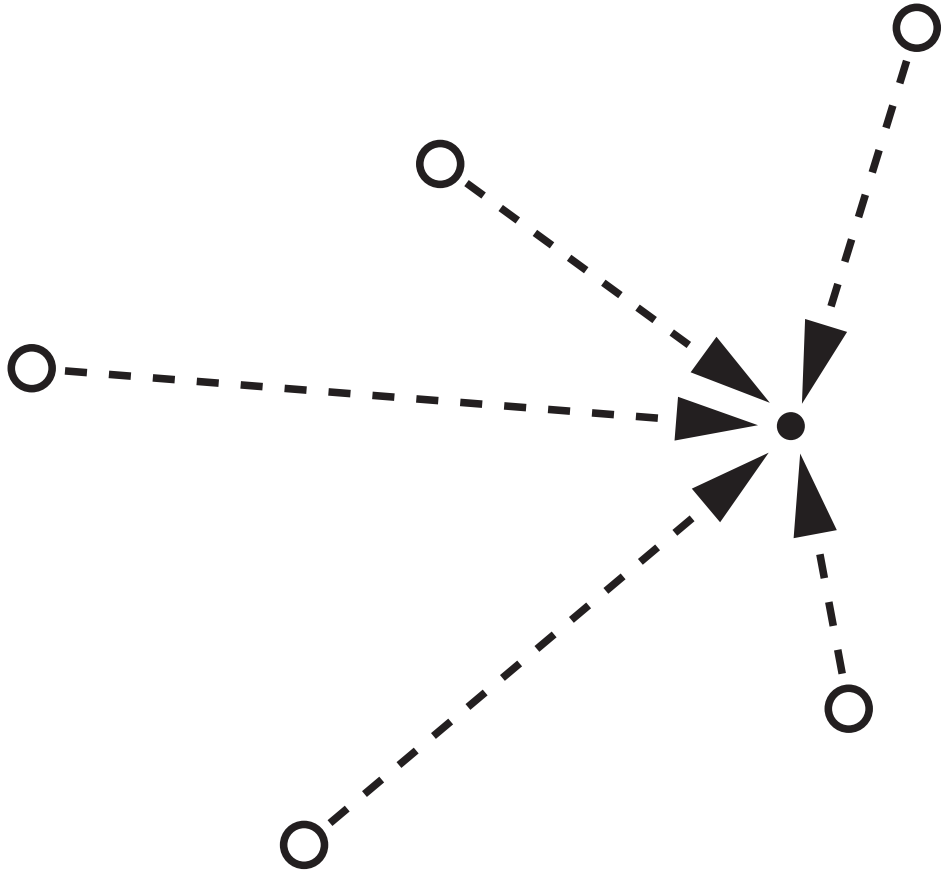
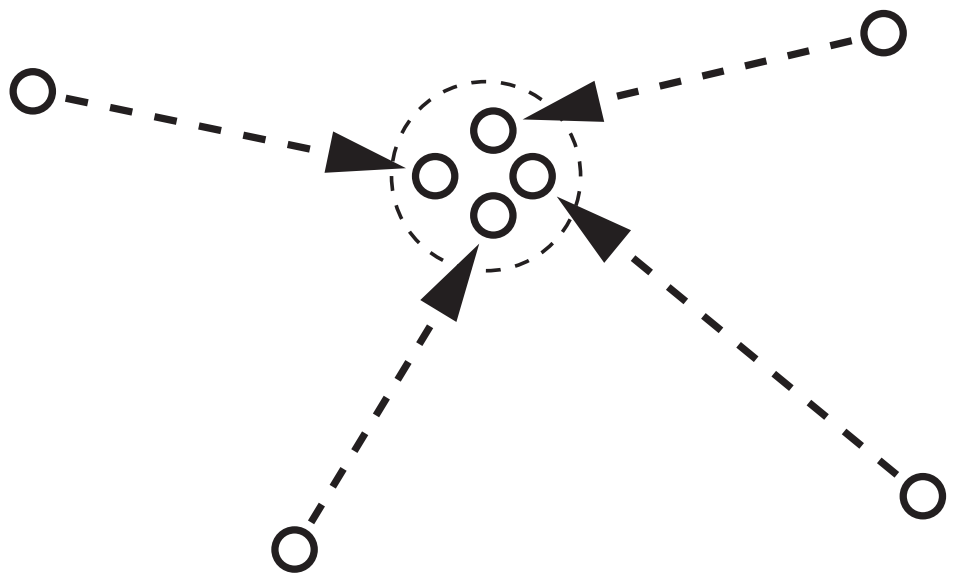


Solvable in CORDA with limited visibility.

Near-Gathering problem

- All robots must gather in a small-enough area.
- Collisions must be avoided.



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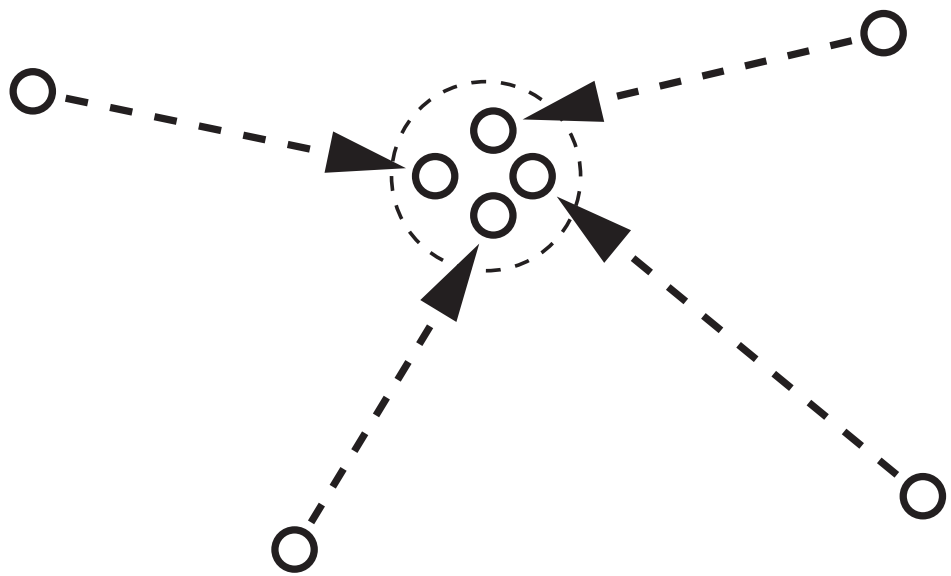
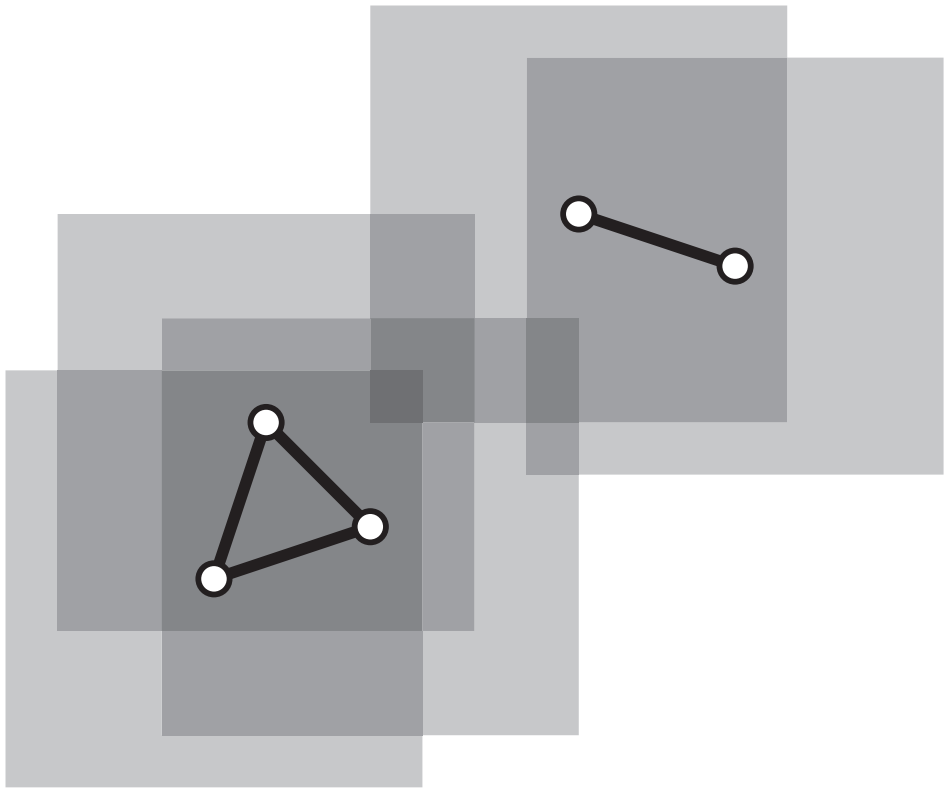


- Solvable in semi-synchronous models with limited visibility.
- Is it solvable in asynchronous CORDA with limited visibility? (This would imply that all problems solvable in full visibility models are also solvable with limited visibility.)

Initial conditions

Let $G(0)$ be the initial *visibility graph*.



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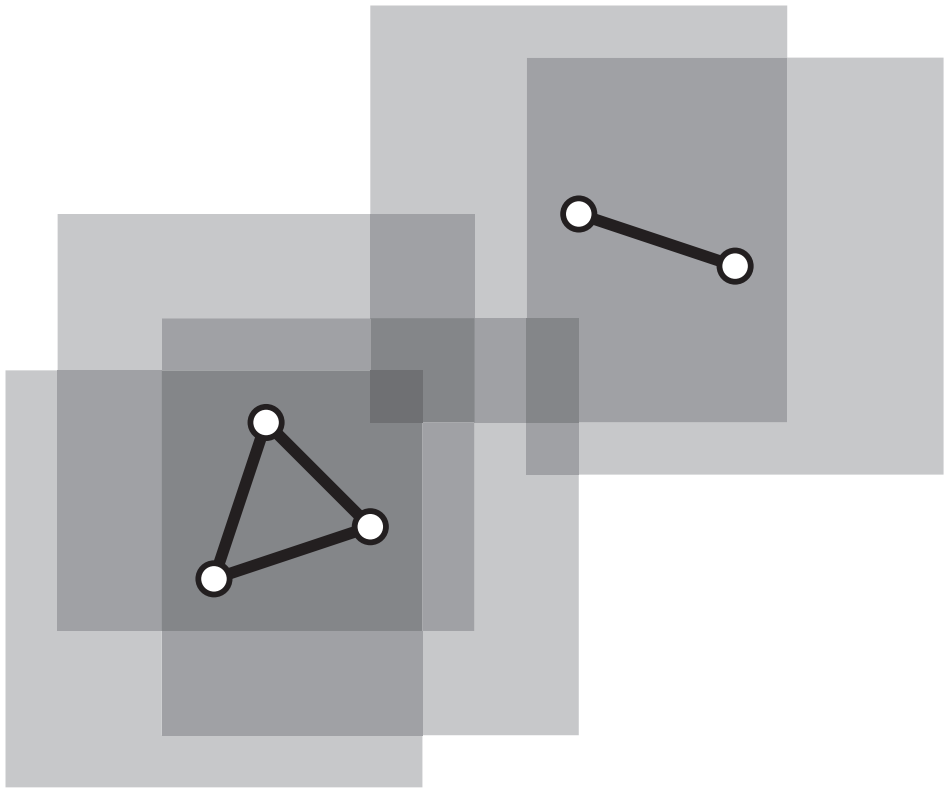
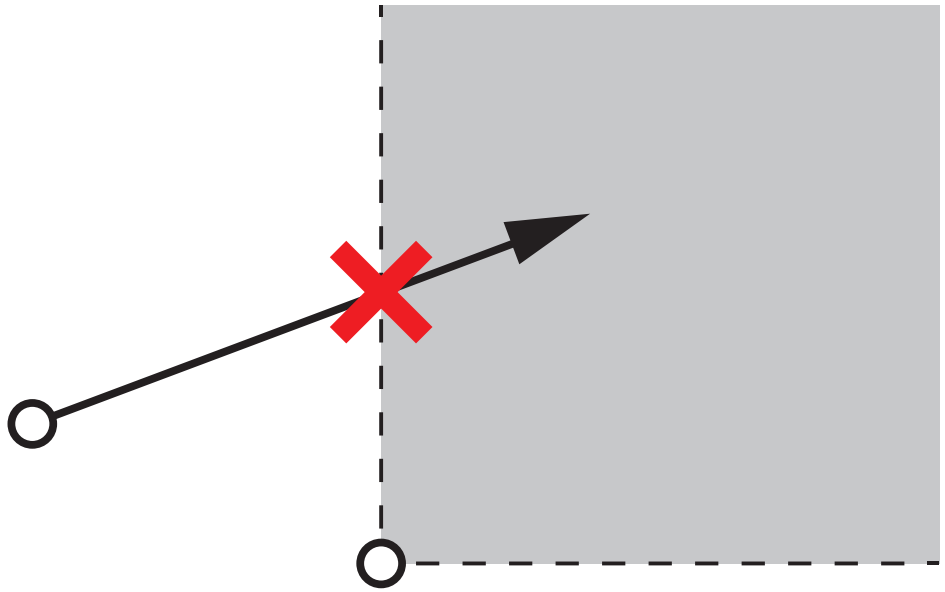


For NEAR-GATHERING to be solvable, $G(0)$ must be connected.

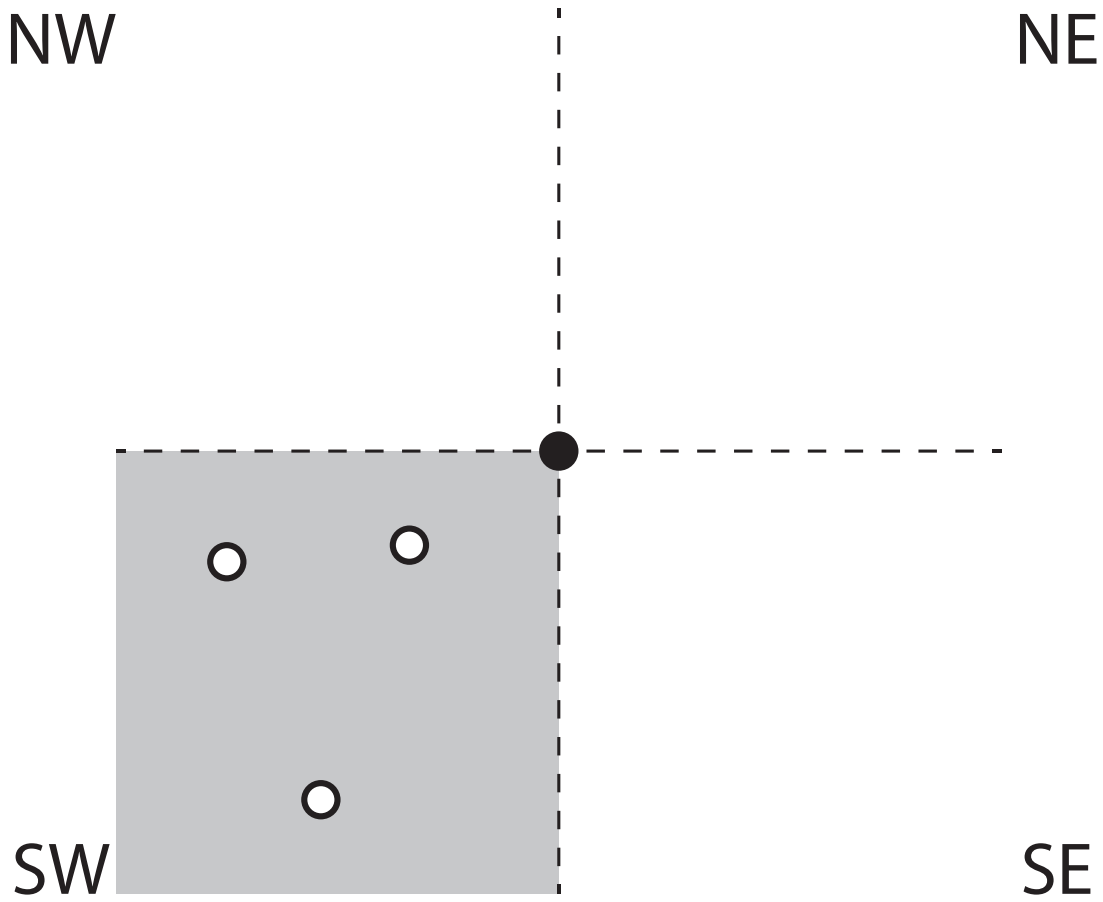
Algorithm guidelines

- Each robot moves only upwards and rightwards.
- No robot willingly enters the "move space" of another robot.
- Robots try to move without losing visibility with each other.
- No robot moves "too much" during a single cycle.



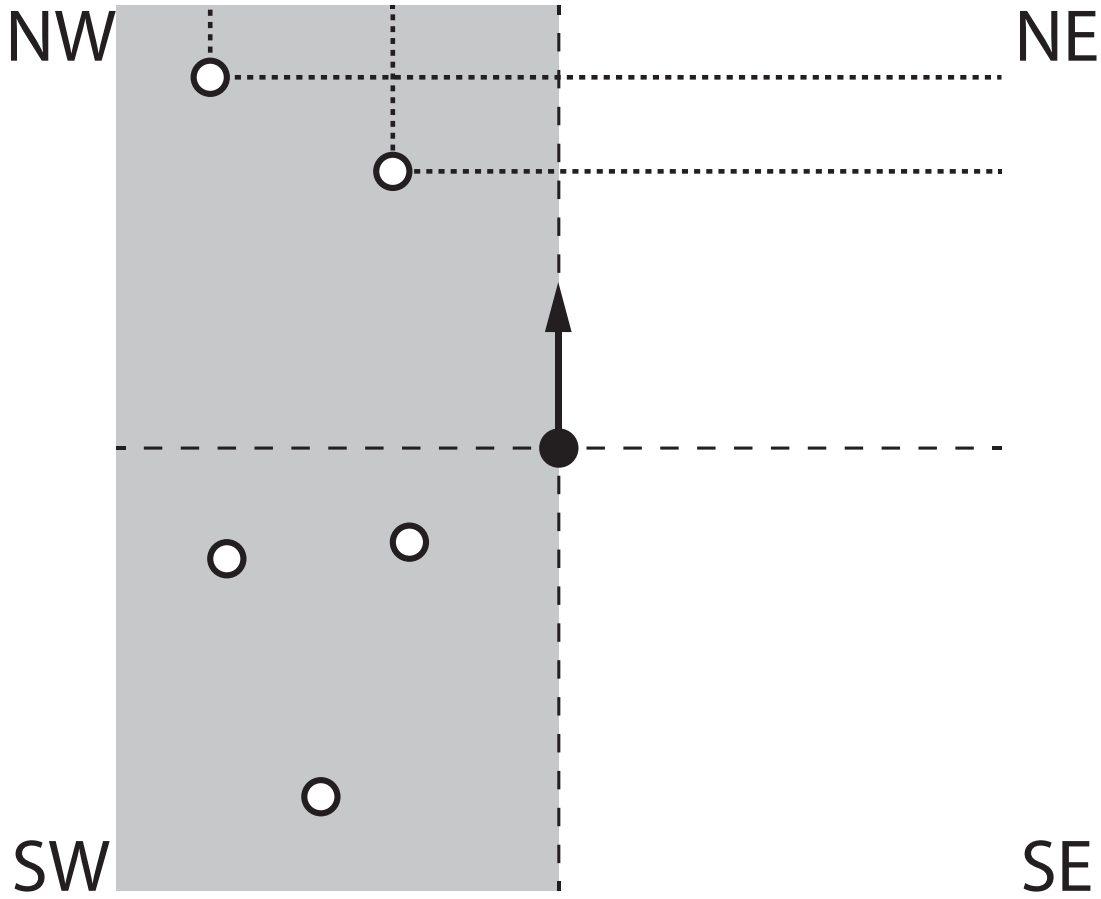
Algorithm

If I see robots only in SW, I do not move.



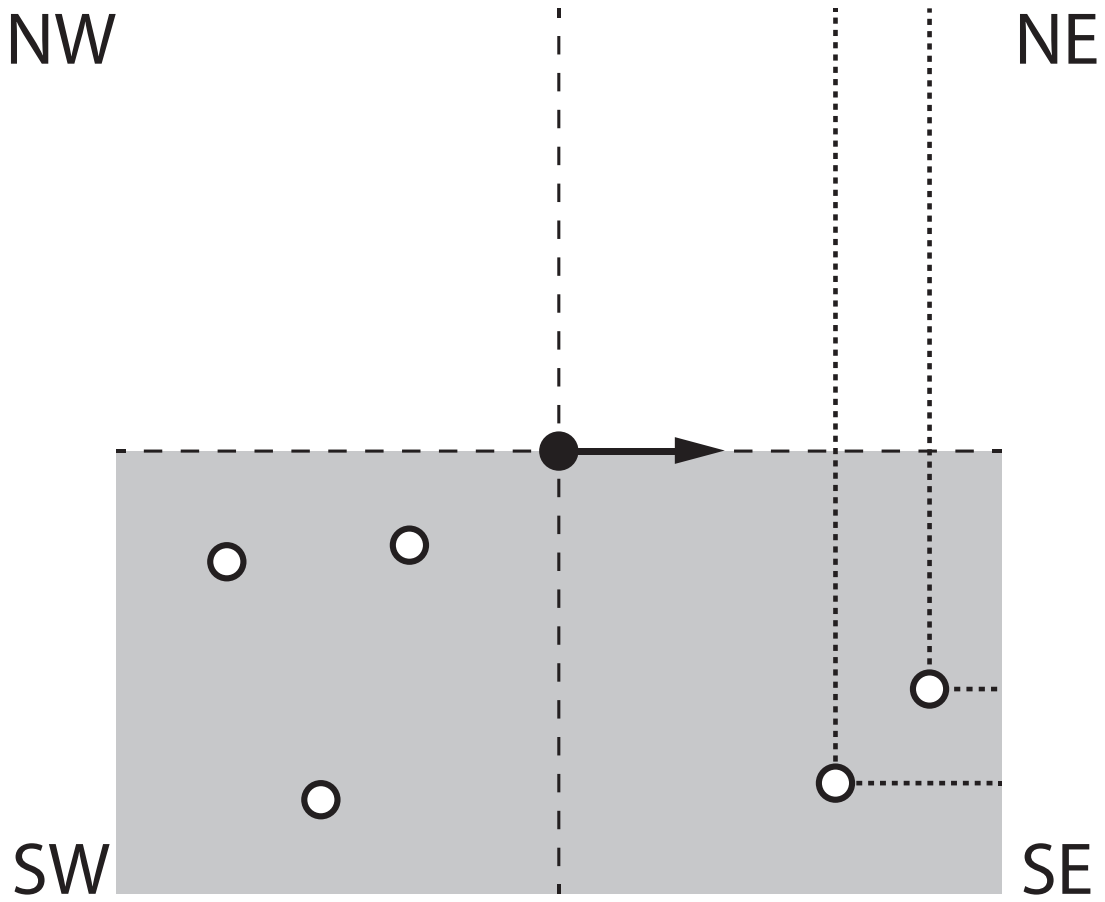
Algorithm

If I see robots only in SW $\cup$ NW, I move North.



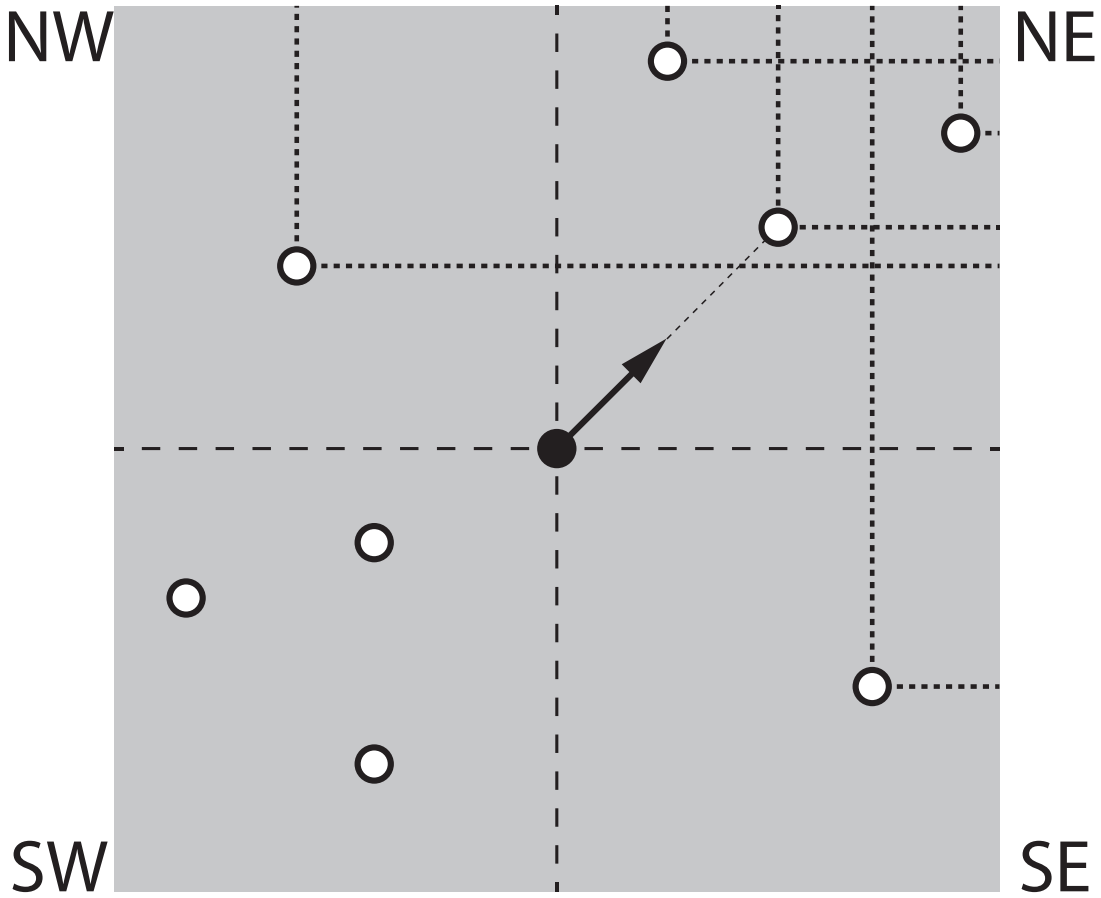
Algorithm

If I see robots only in SW $\cup$ SE, I move East.



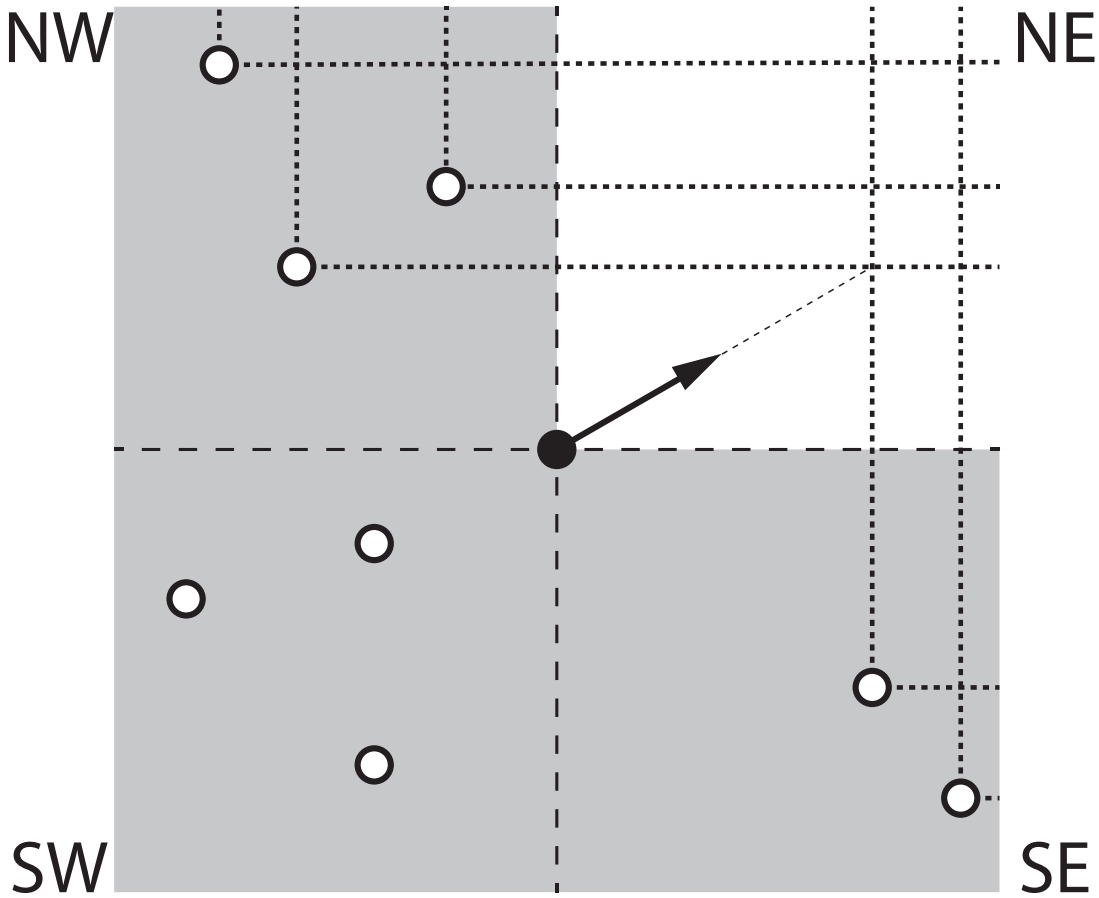
Algorithm

If I see some robots in NE, I move toward the nearest one.



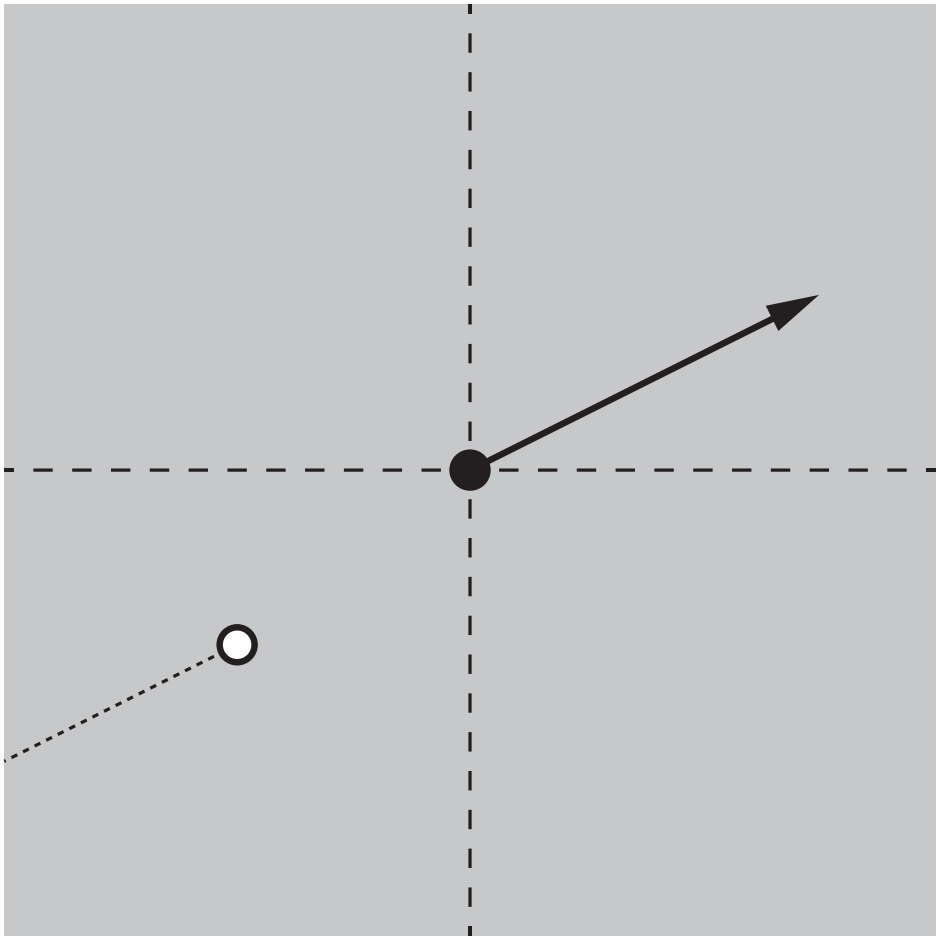
Algorithm

Otherwise (I see robots in NW and SE, possibly in SW, but no robot in NE)



Algorithm

While computing my destination point, I make sure that I do not lose visibility with any other robot that I currently see.



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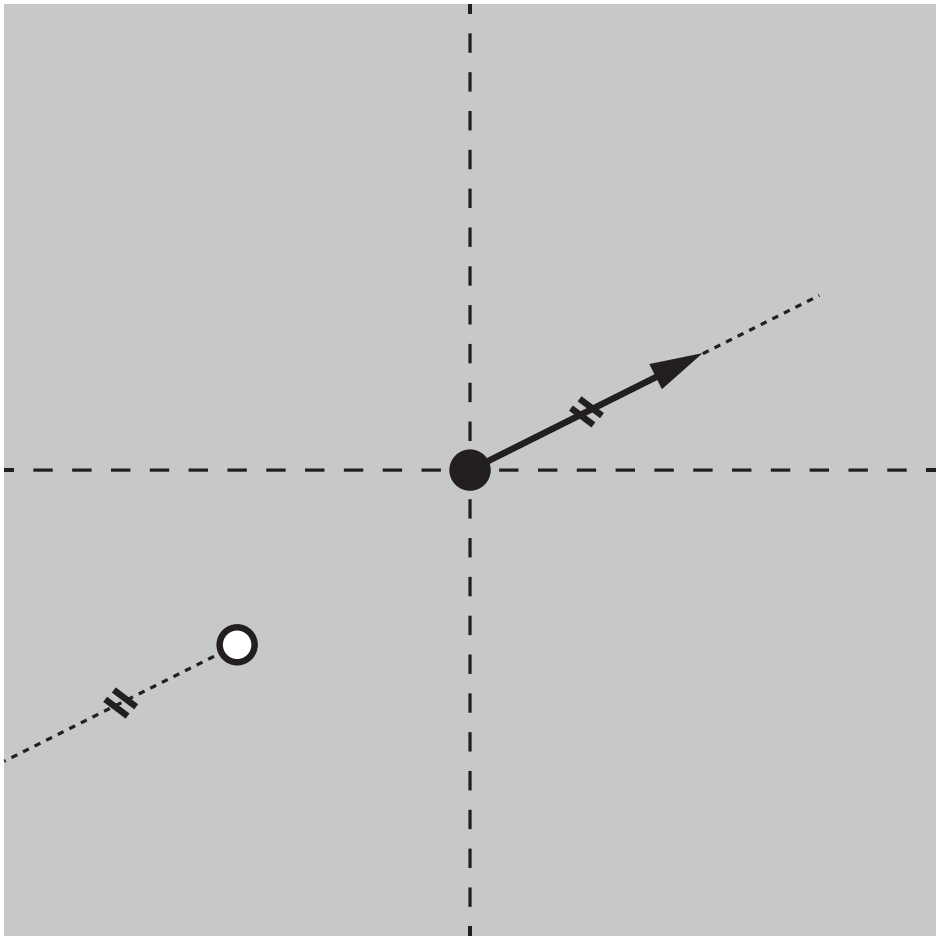

Algorithm correctness

Proof sketch:

- The visibility graph remains connected.
- No collision occurs.
- The robots converge to the same point.

Mutual awareness

Definition

Robots r and s are *mutually aware* at time t if r saw s during its last Look phase, and vice versa.

Lemma

If r and s are mutually aware at time t, they are mutually aware at any time $t' > t$.

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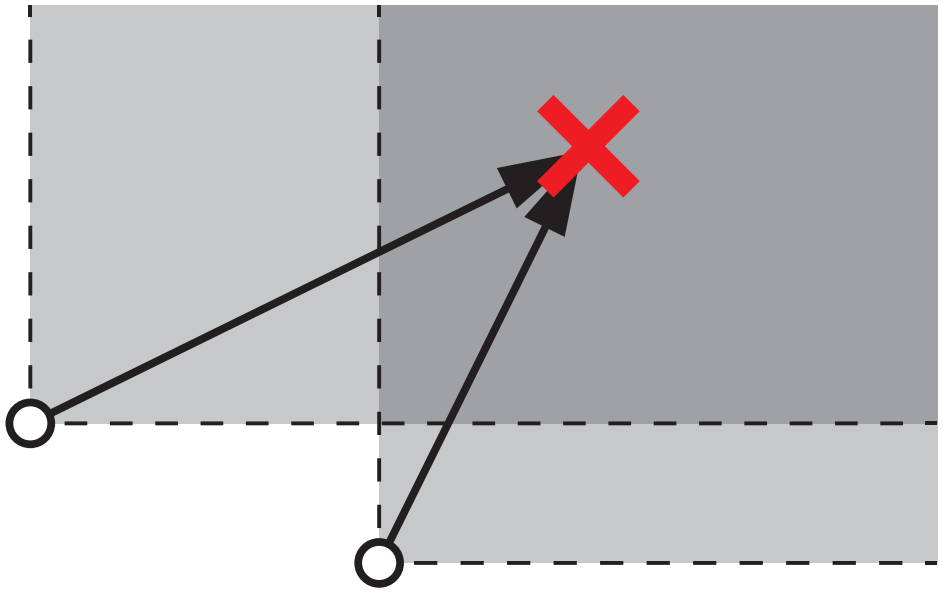
Corollary

At any time t, the visibility graph $G(t)$ is a supergraph of $G(0)$. Hence $G(t)$ is connected.

Collision avoidance

Collisions never occur because

- robots move by small-enough steps,
- hence they must become mutually aware before colliding,
- no robot willingly enters another robot's move space
- (the actual distance function is not relevant here)



Convergence

Each robot's coordinates are monotonically increasing and bounded from above, hence each robot has a *convergence point*.

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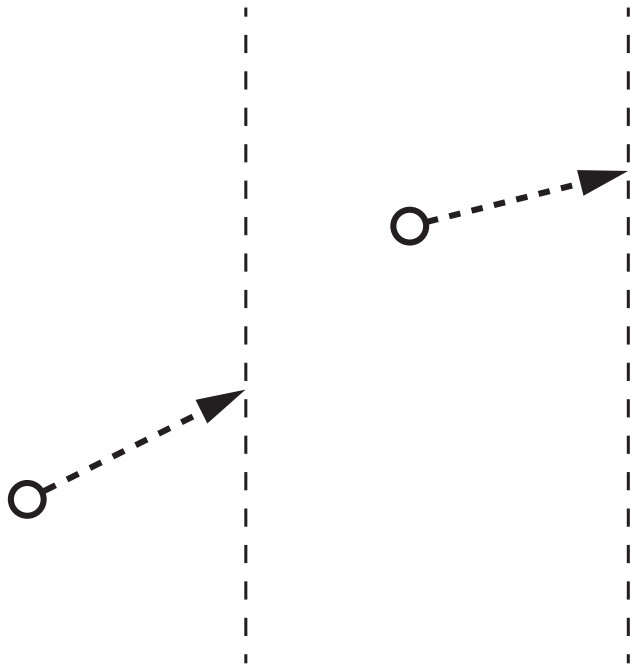
All robots have the same convergence point.



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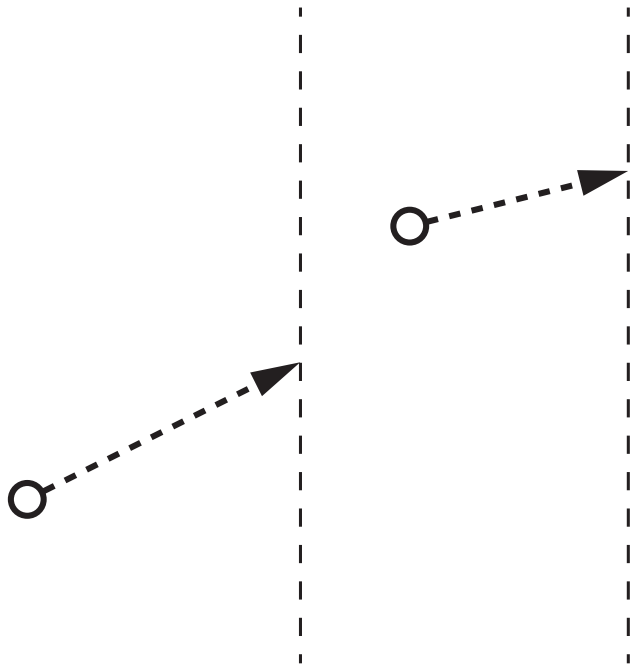

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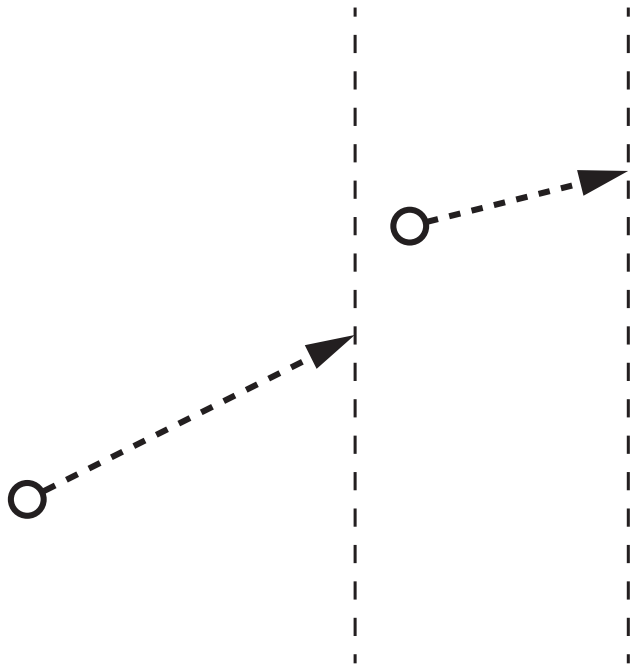
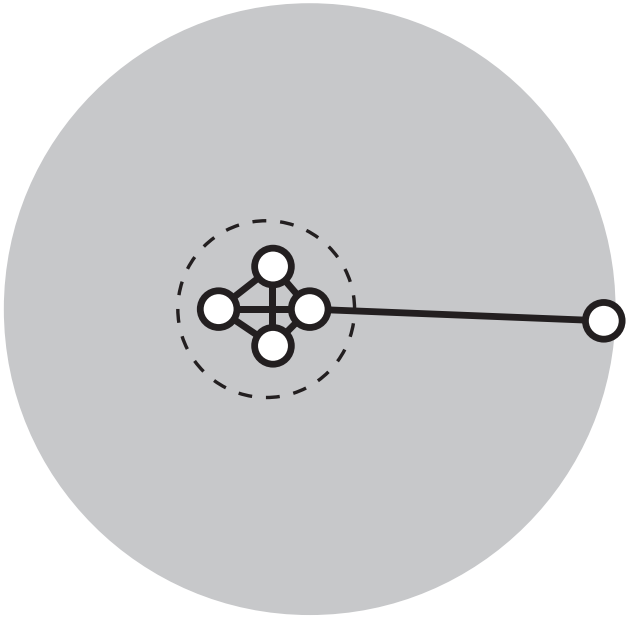


Naive termination

How do robots know when to terminate?
Without further assumptions on the model, the termination problem is unsolvable.



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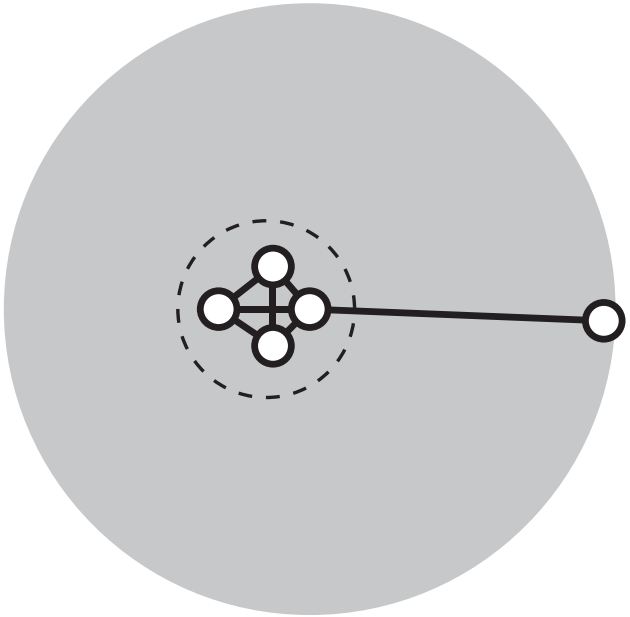

A simple way to solve it is to let the robots know their number, n.
Whenever a robot sees $n - 1$ other robots in a small-enough neighborhood, it terminates.

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Let us provide each robot with a *light*, which can be turned **on** or **off**, and can be seen by nearby robots.

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Termination protocol:

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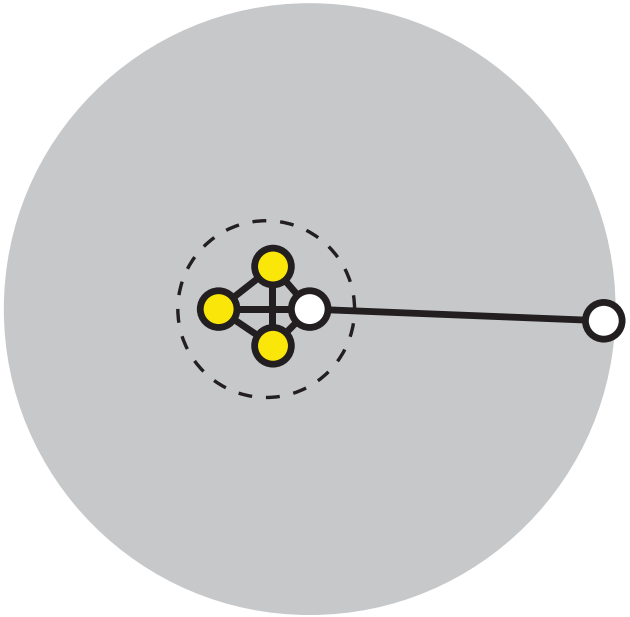

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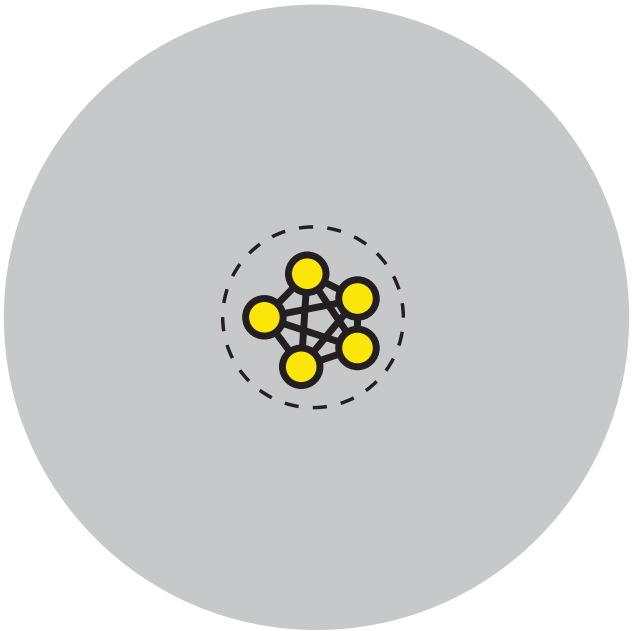

Conclusions

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- The total axis agreement assumption is quite strong.
  Can it be weakened?