Partial Searchlight Scheduling is Strongly PSPACE-complete

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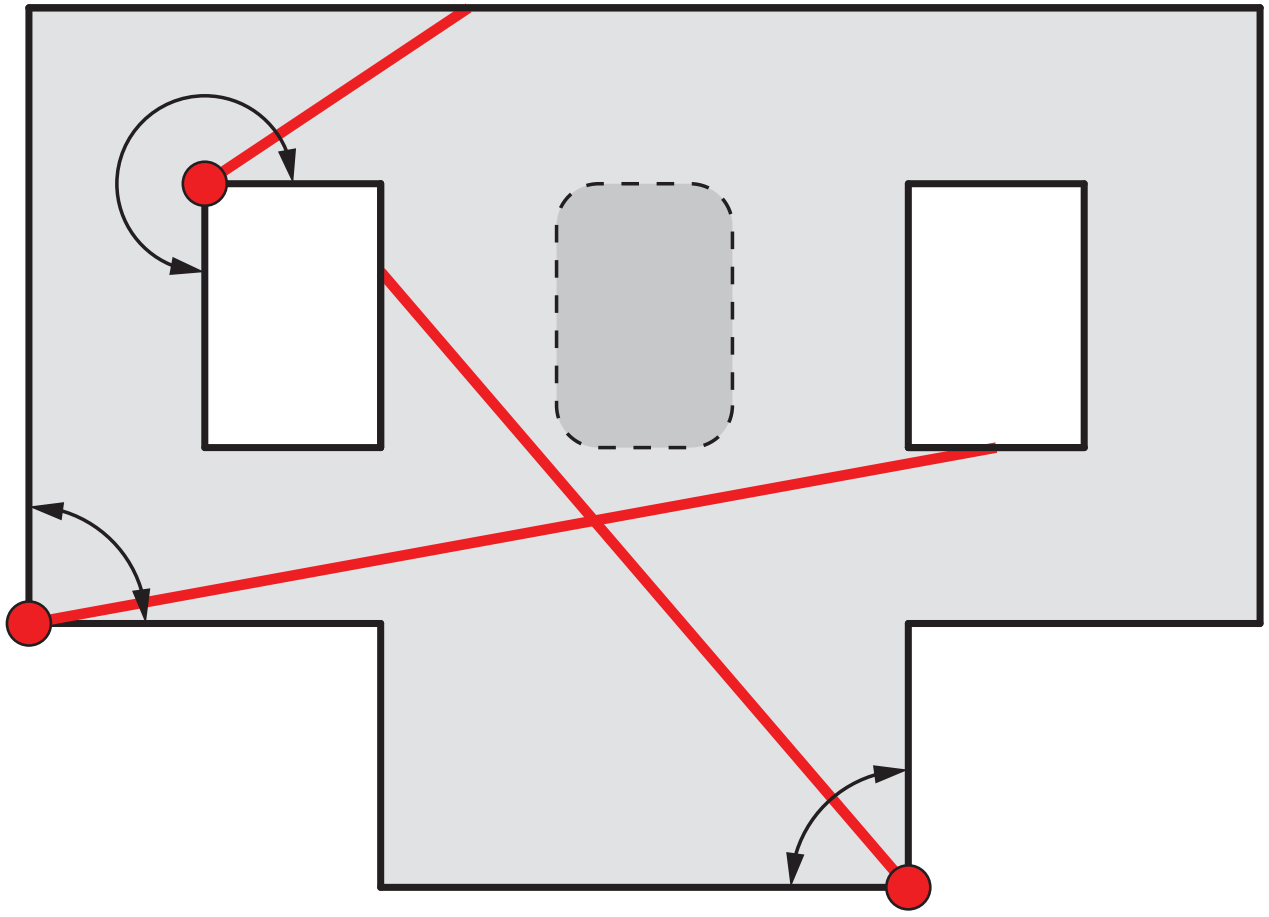
Partial Searchlight Scheduling Problem

- Can the three guards keep any moving intruder out of the dashed area by turning their searchlights?



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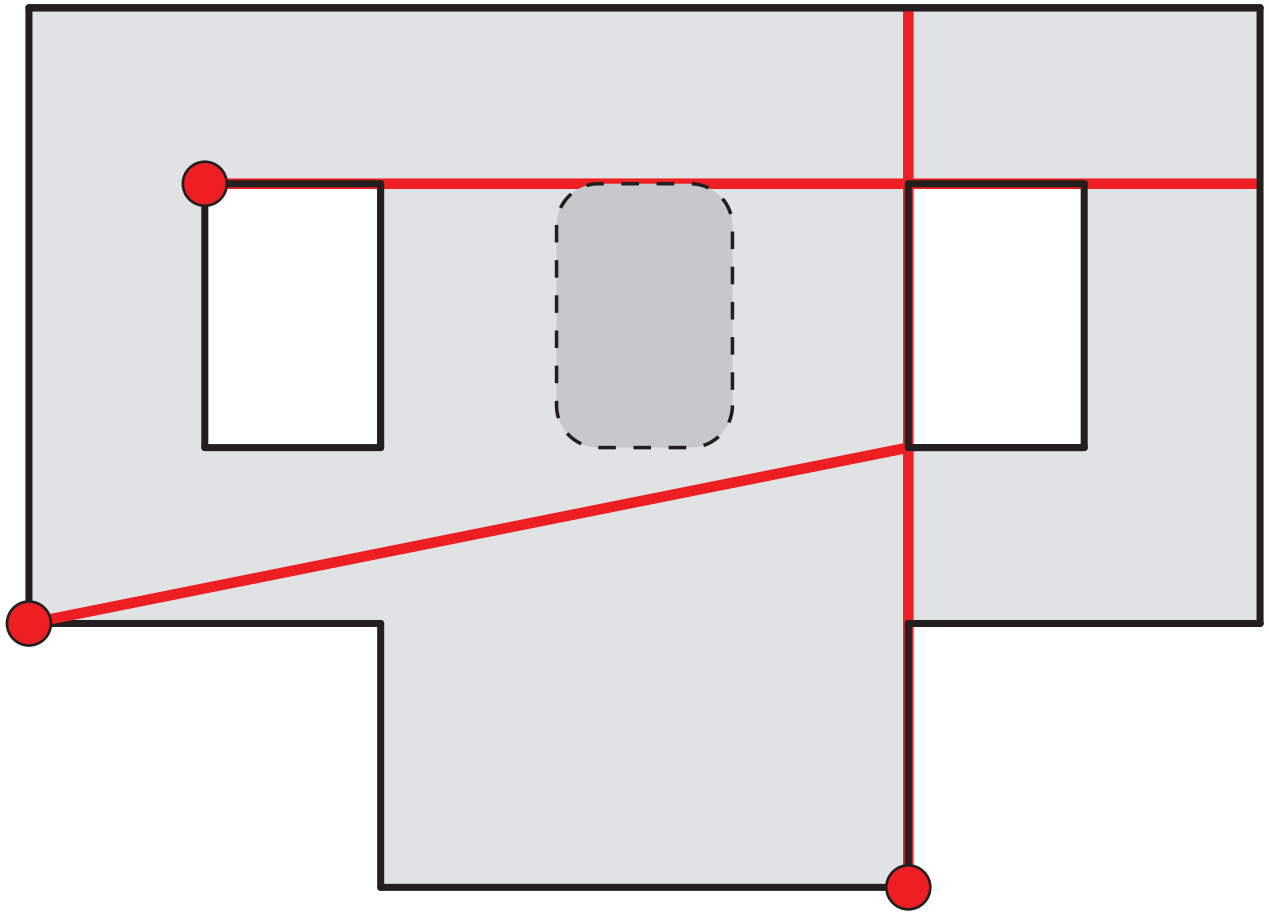

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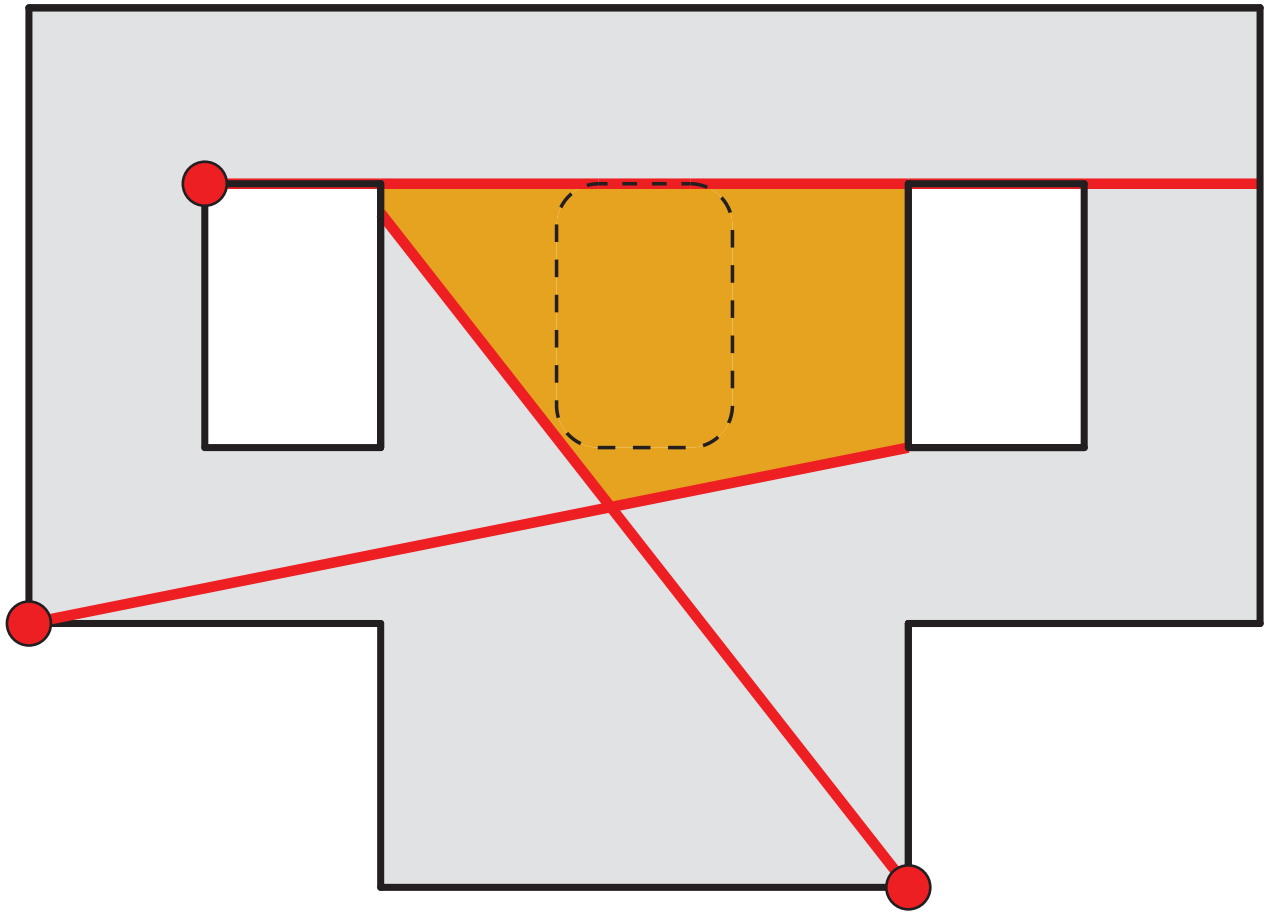

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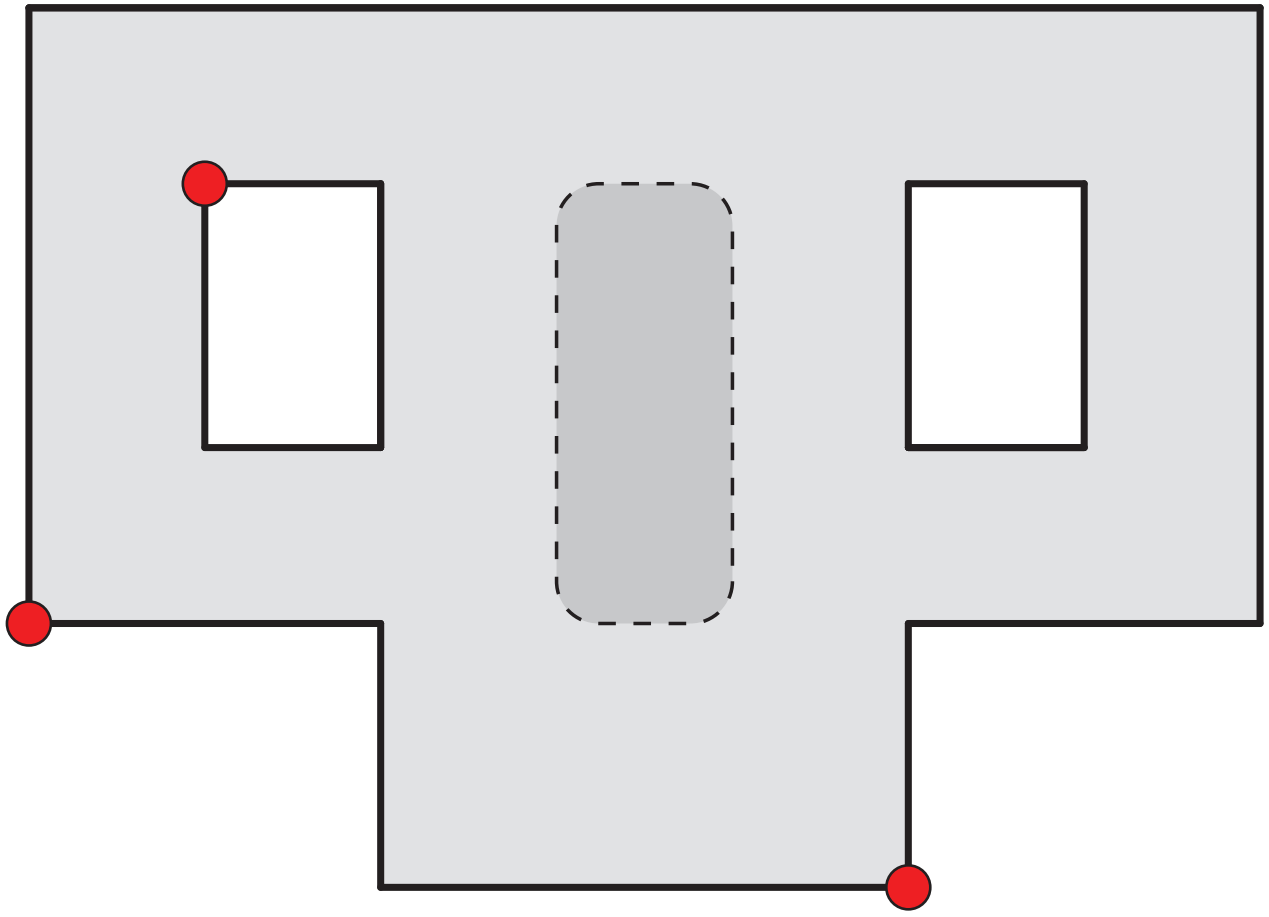

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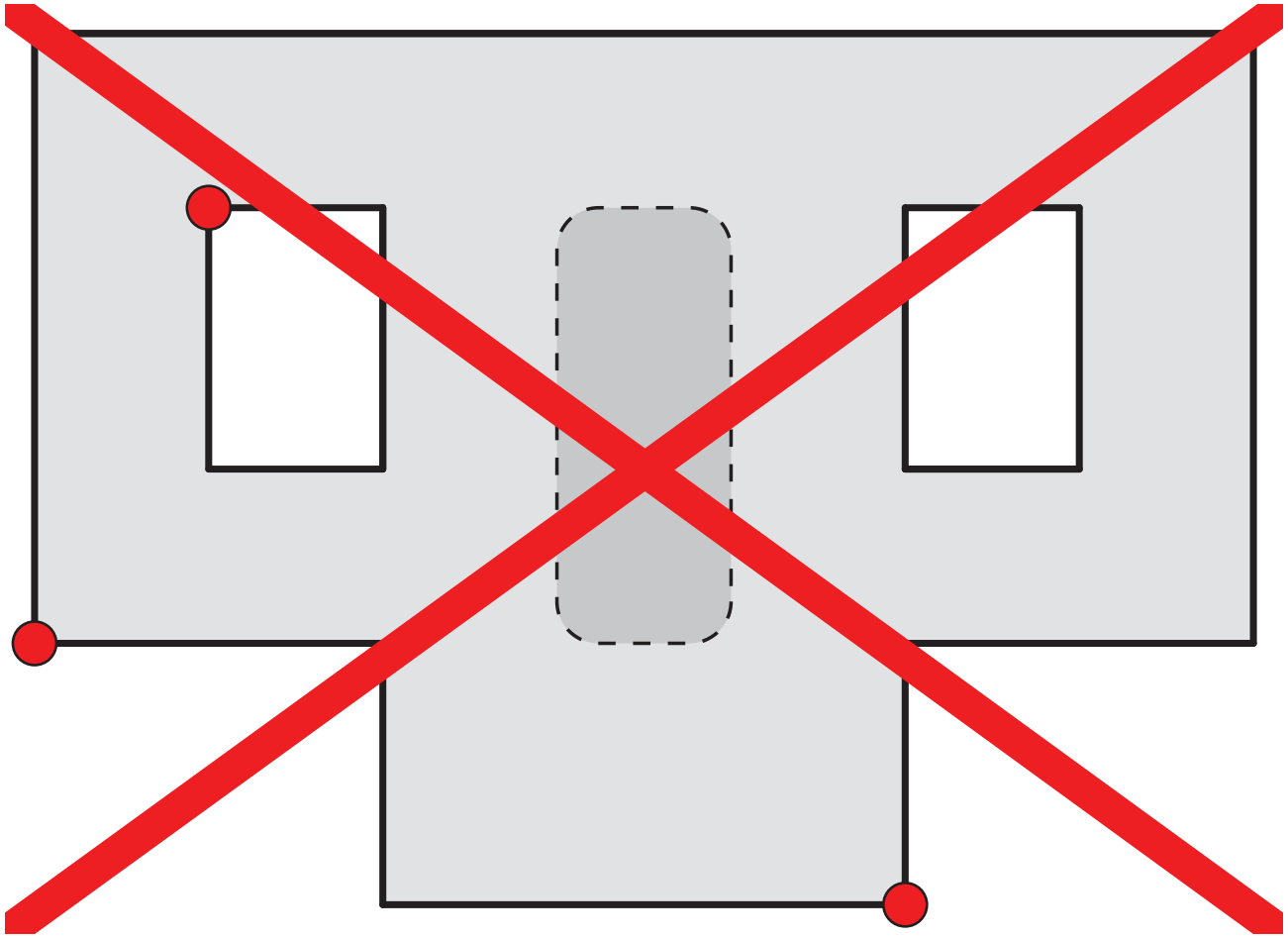
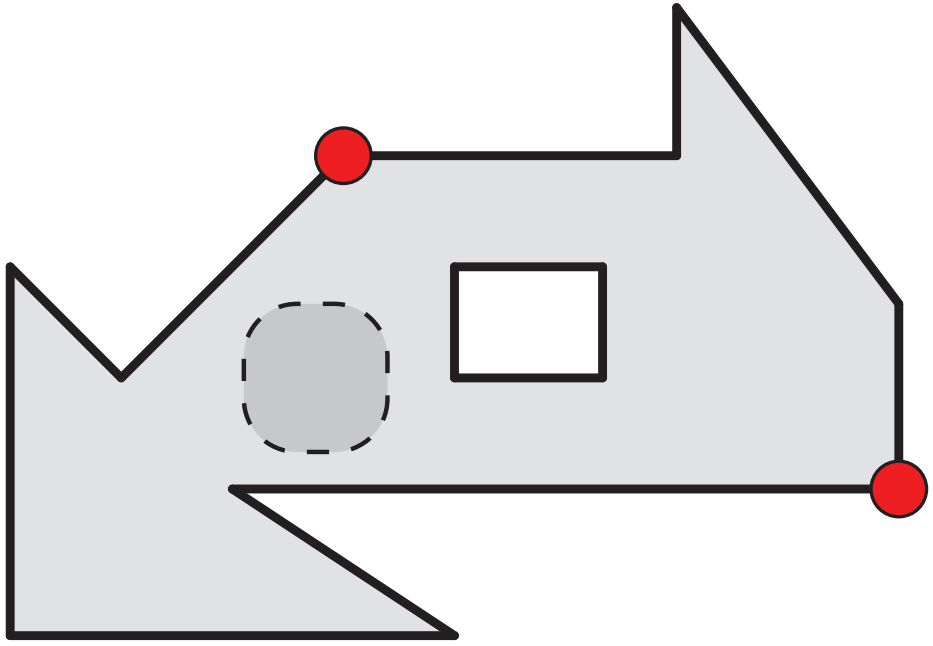


Complexity of Partial Searchlight Scheduling

- Deciding partial searchability is in **PSPACE**.



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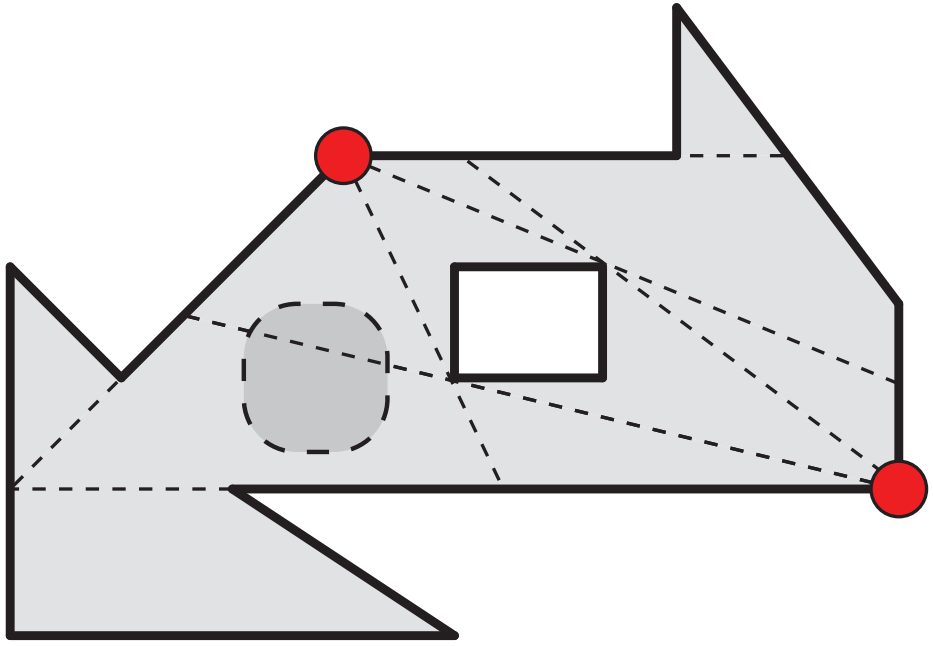

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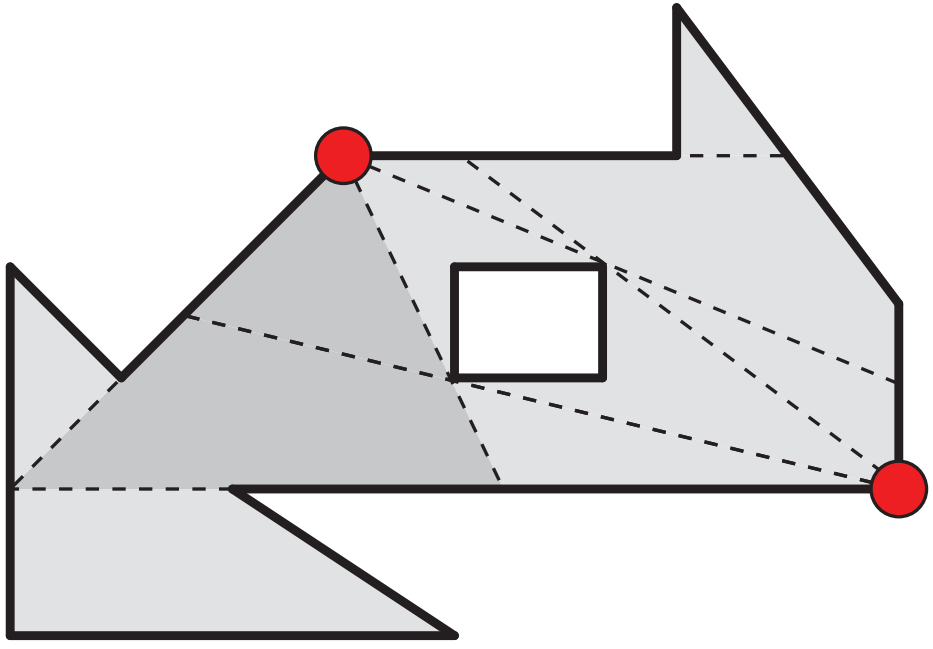

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- Extend the target area to a set of cells.

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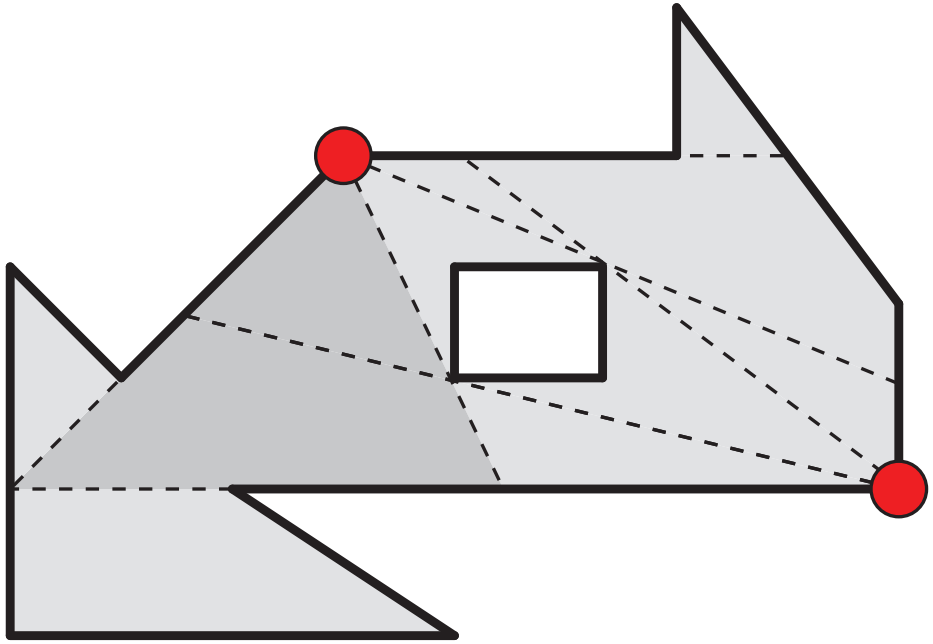

- Identify *critical angles* and split the polygon into *cells*.
- Extend the target area to a set of cells.
- Search the target area by turning one searchlight at a time, and stopping only at critical angles.

Nondeterministic Constraint Logic machines

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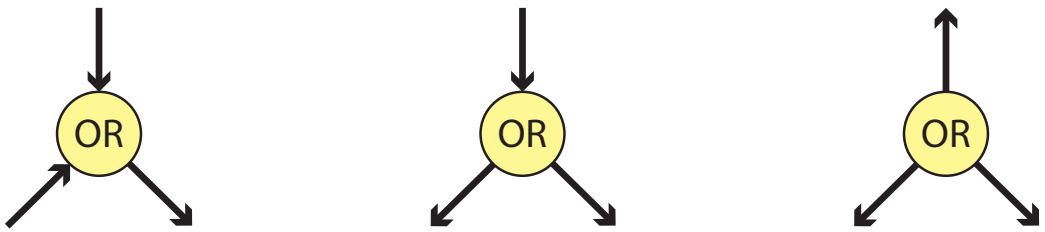

YES YES NO

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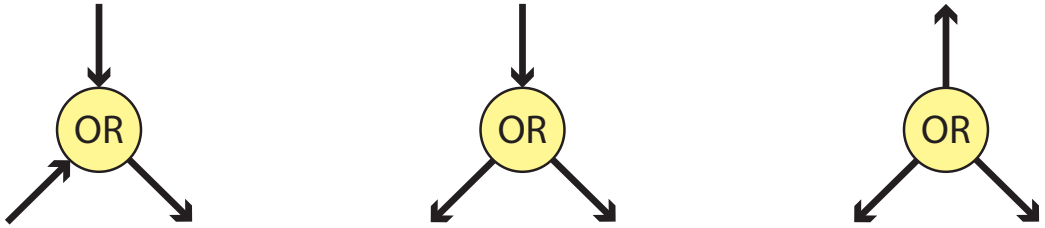

YES YES NO

  - **AND node.** Either the thick edge is directed inward, or both thin edges are directed inward.

YES YES YES NO

Nondeterministic Constraint Logic machines

- A *legal move* is the reversal of an edge, preserving the above constraints.
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- Each edge has a *target orientation*.

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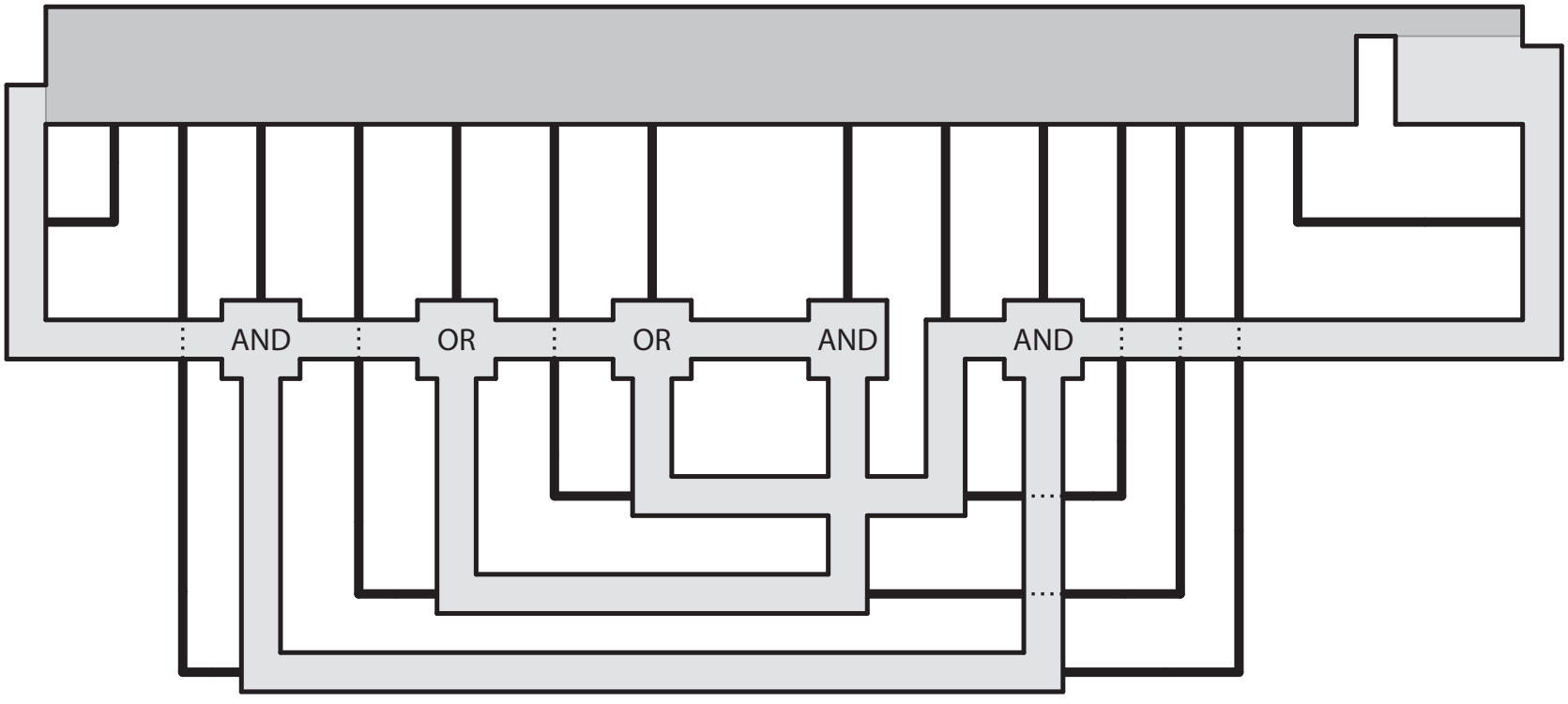
- Moves may be *asynchronous*.

- NCL machines have two *distinguished edges*, e_a and e_b.
- Each edge has a *target orientation*.

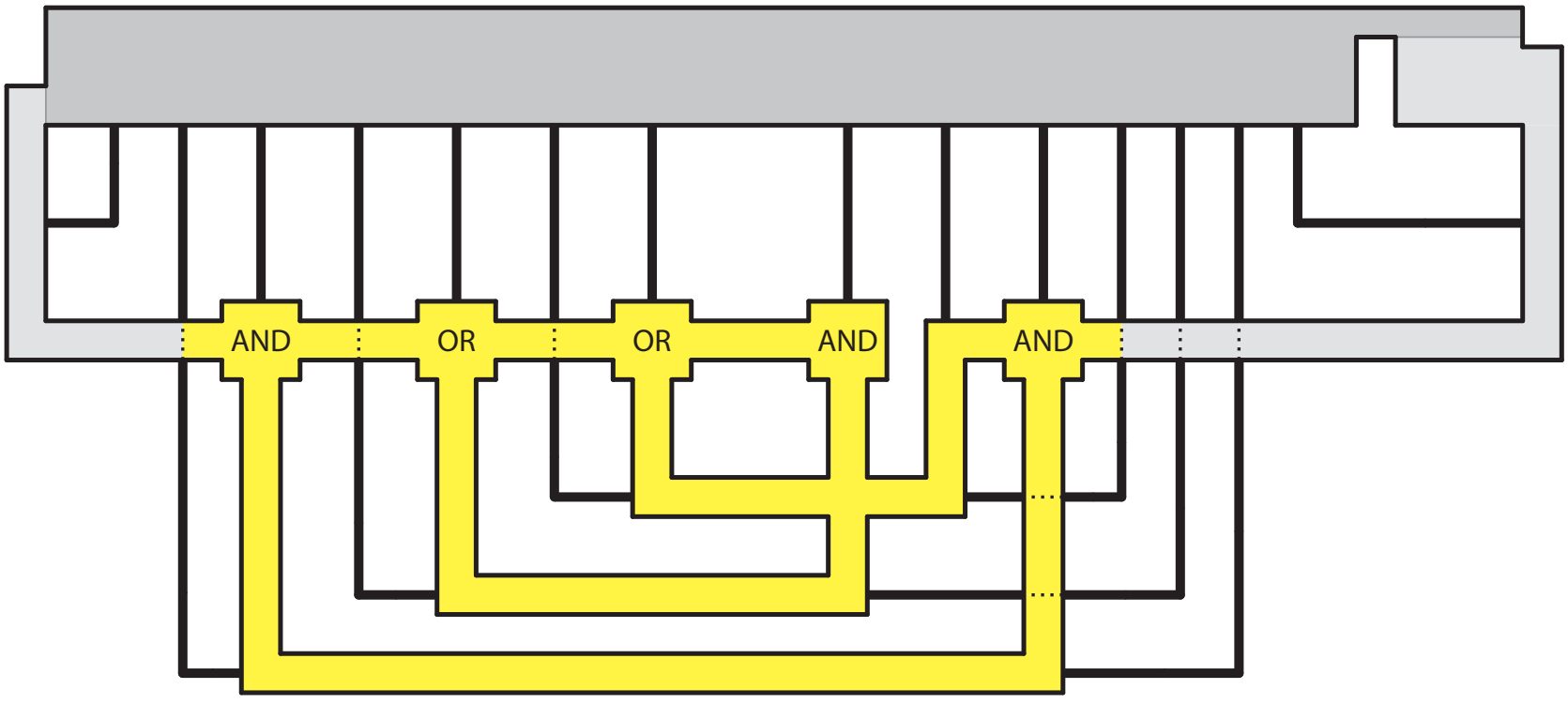
- It is **PSPACE**-complete to decide if legal configurations A and B exist, such that:
  - e_a is in its target orientation in A;
  - e_b is in its target orientation in B;
  - there is a legal sequence of asynchronous moves from A to B.

From NCL machines to orthogonal polygons



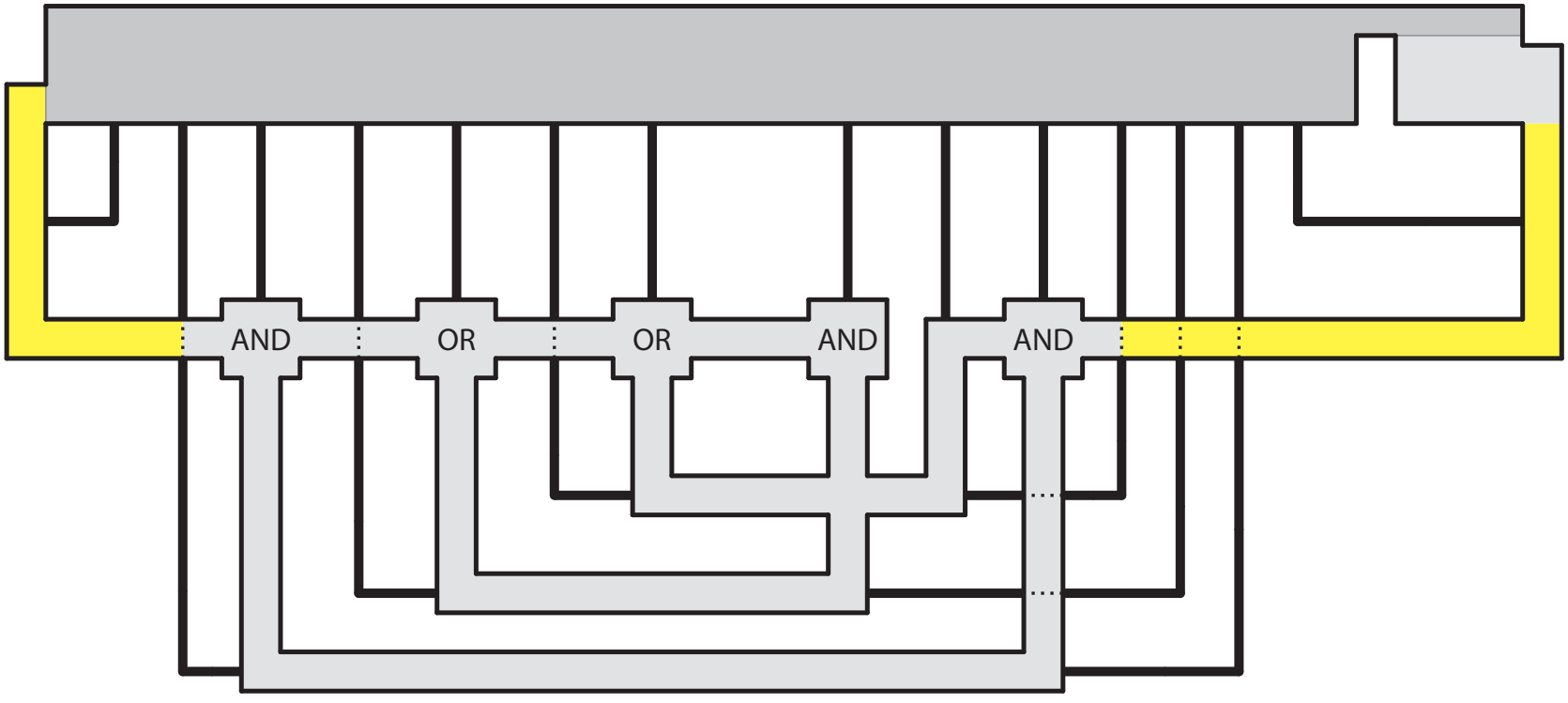
- Reduction overview

From NCL machines to orthogonal polygons



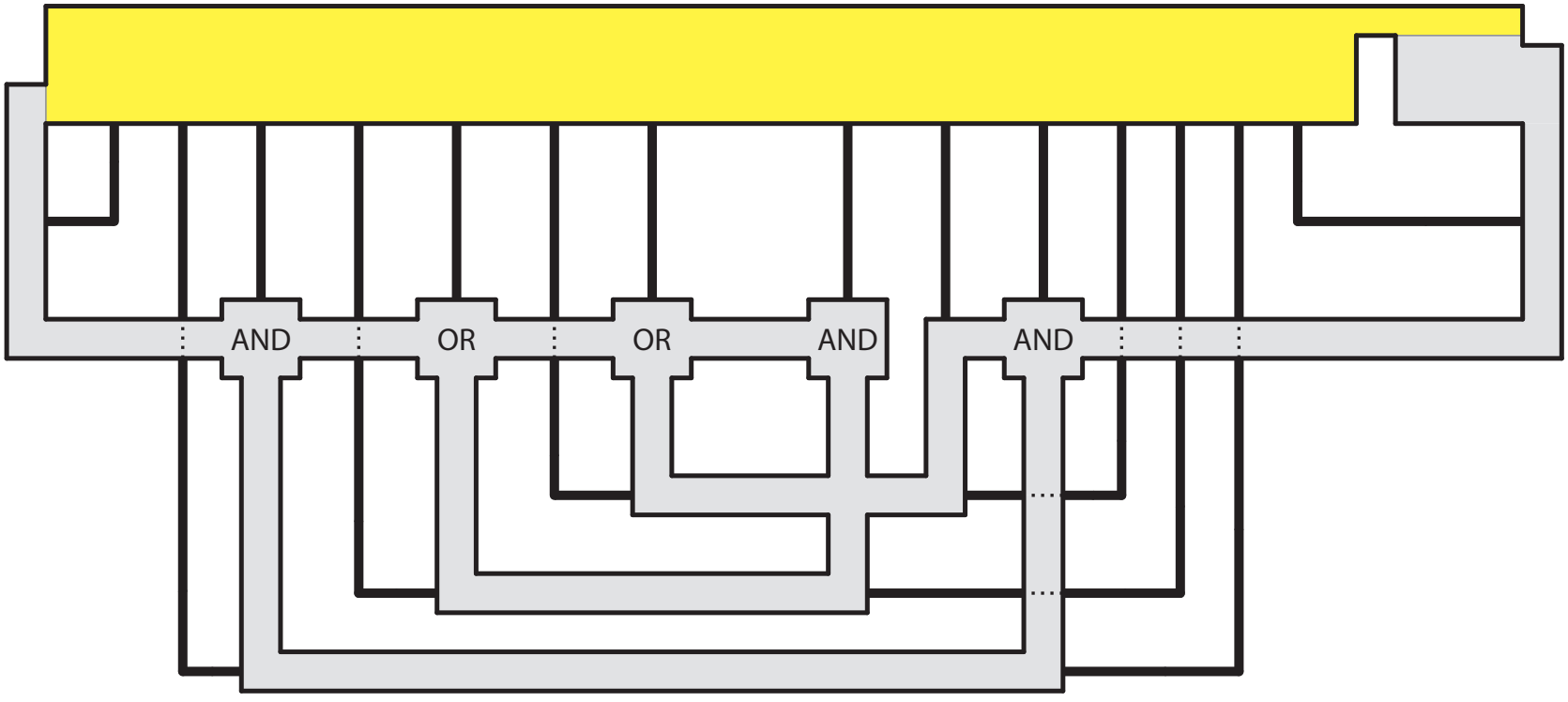
- NCL network (each "segment" contains a guard)

From NCL machines to orthogonal polygons



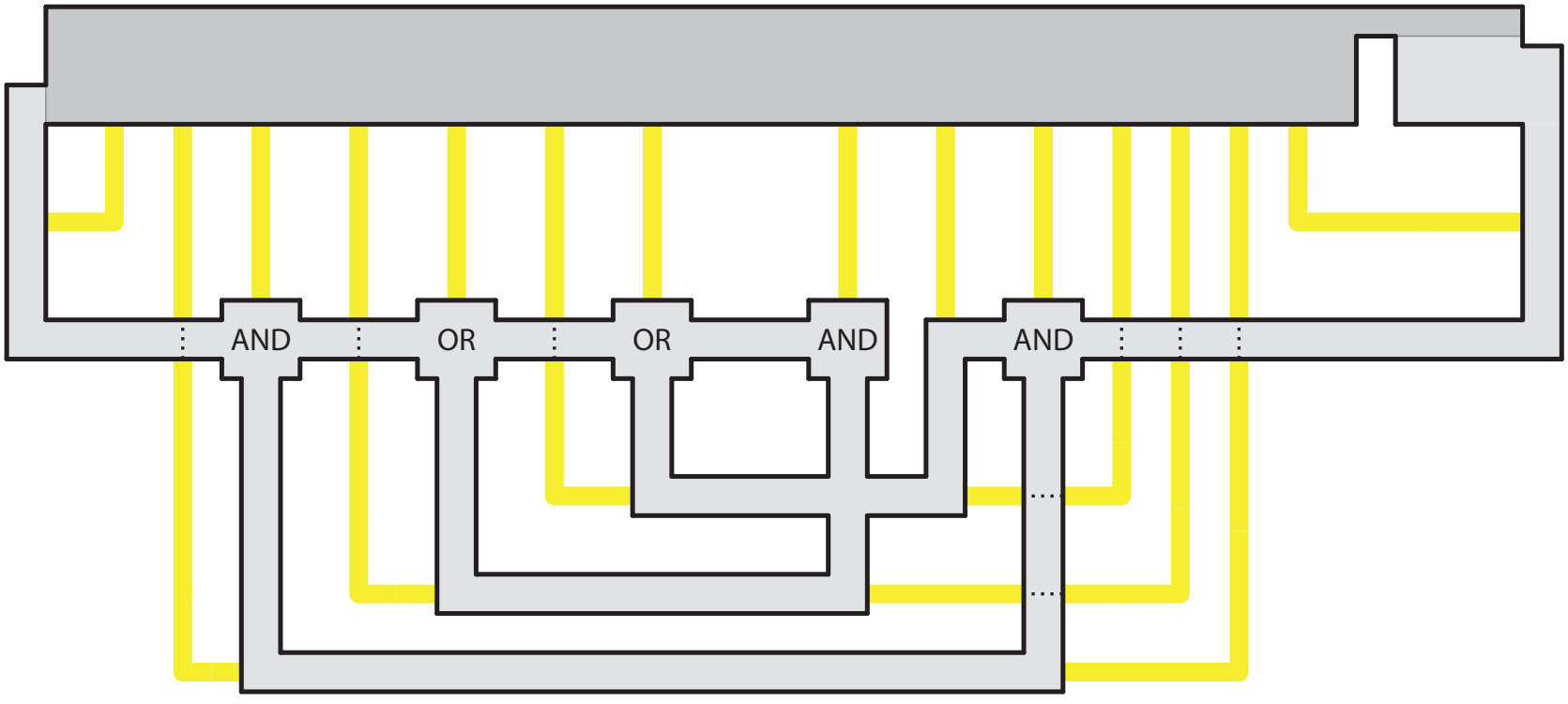
- Distinguished edges

From NCL machines to orthogonal polygons



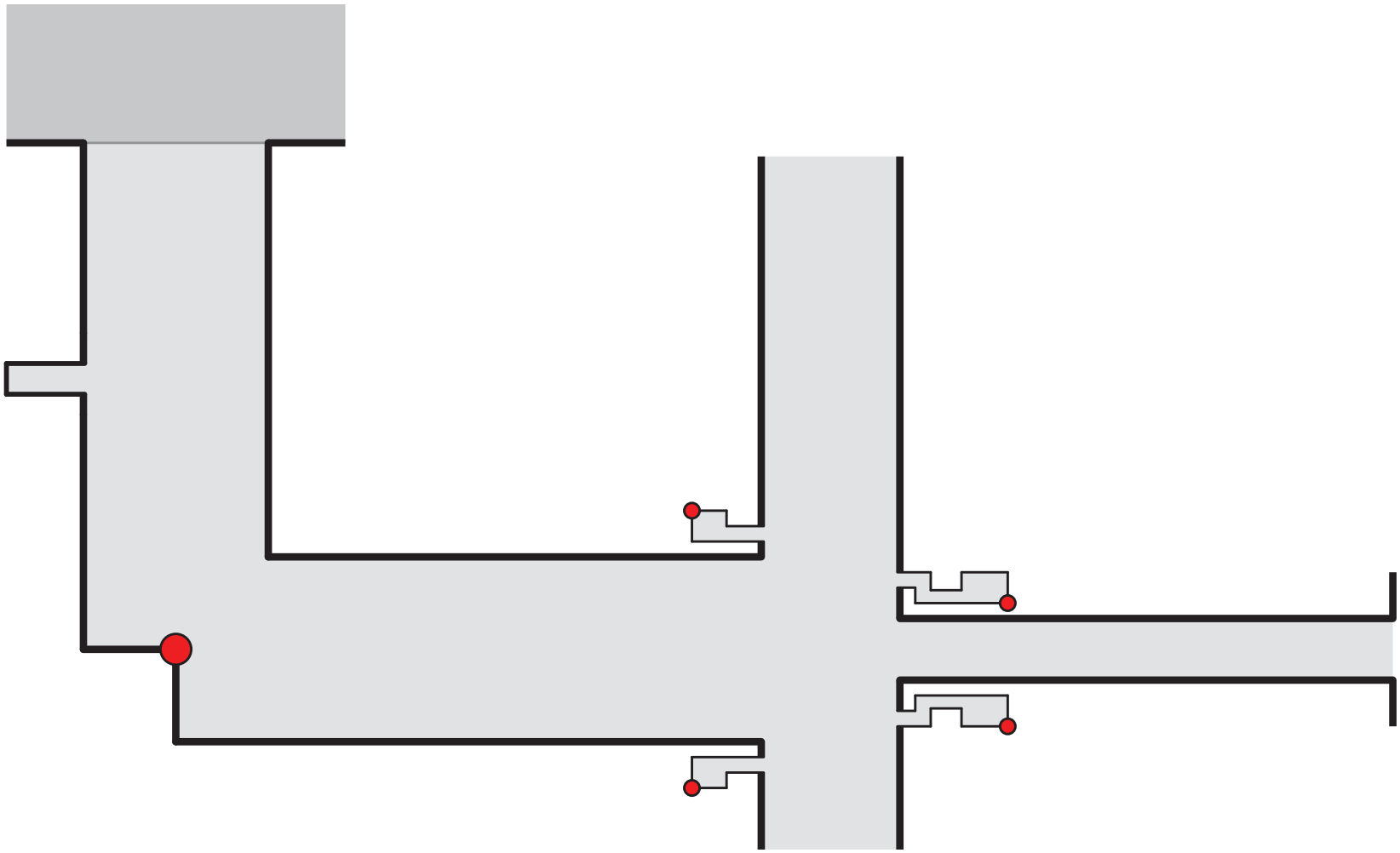
- Target area

From NCL machines to orthogonal polygons



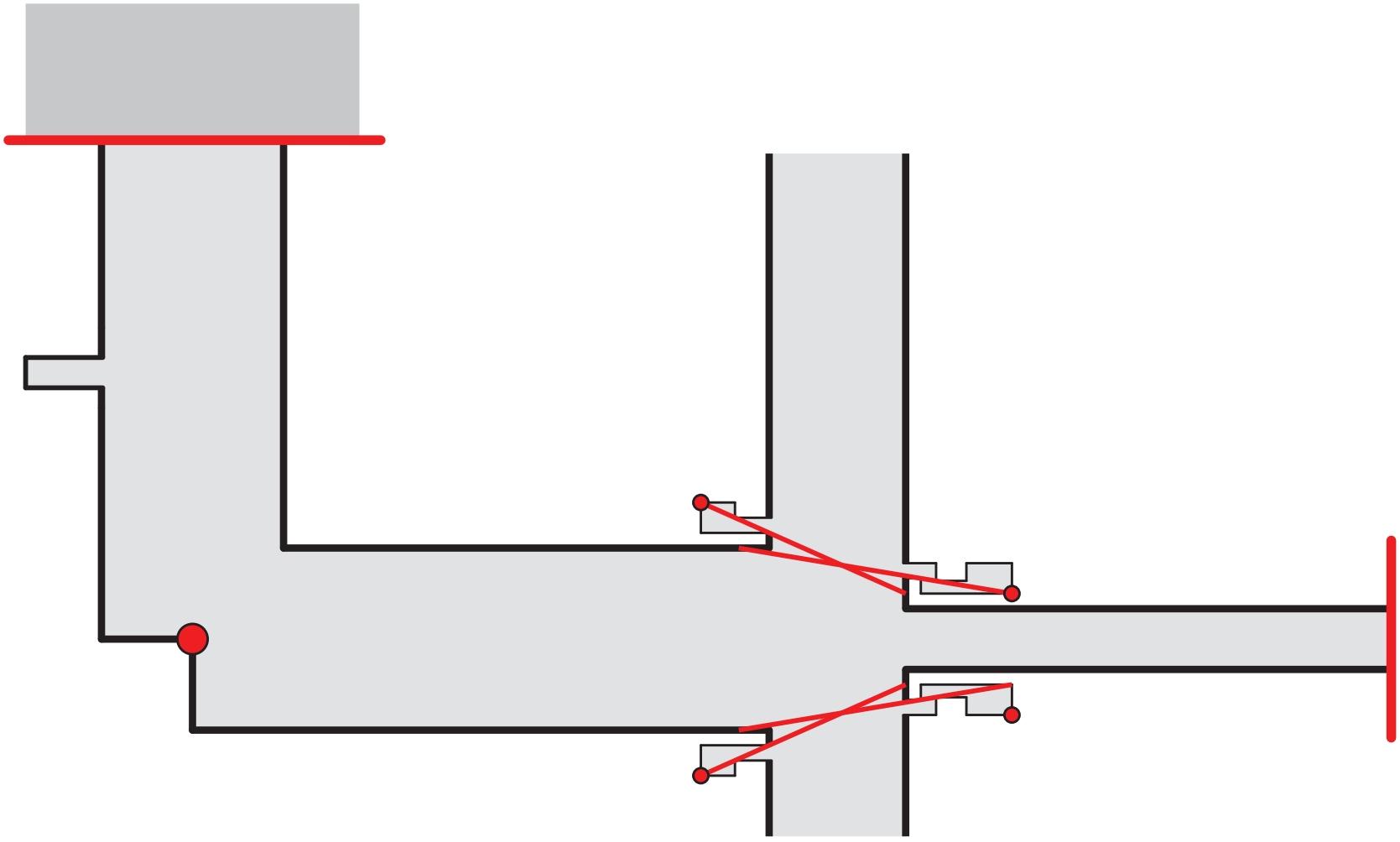
- Pipes (each one contains a guard)

Crossings between pipes and NCL edges



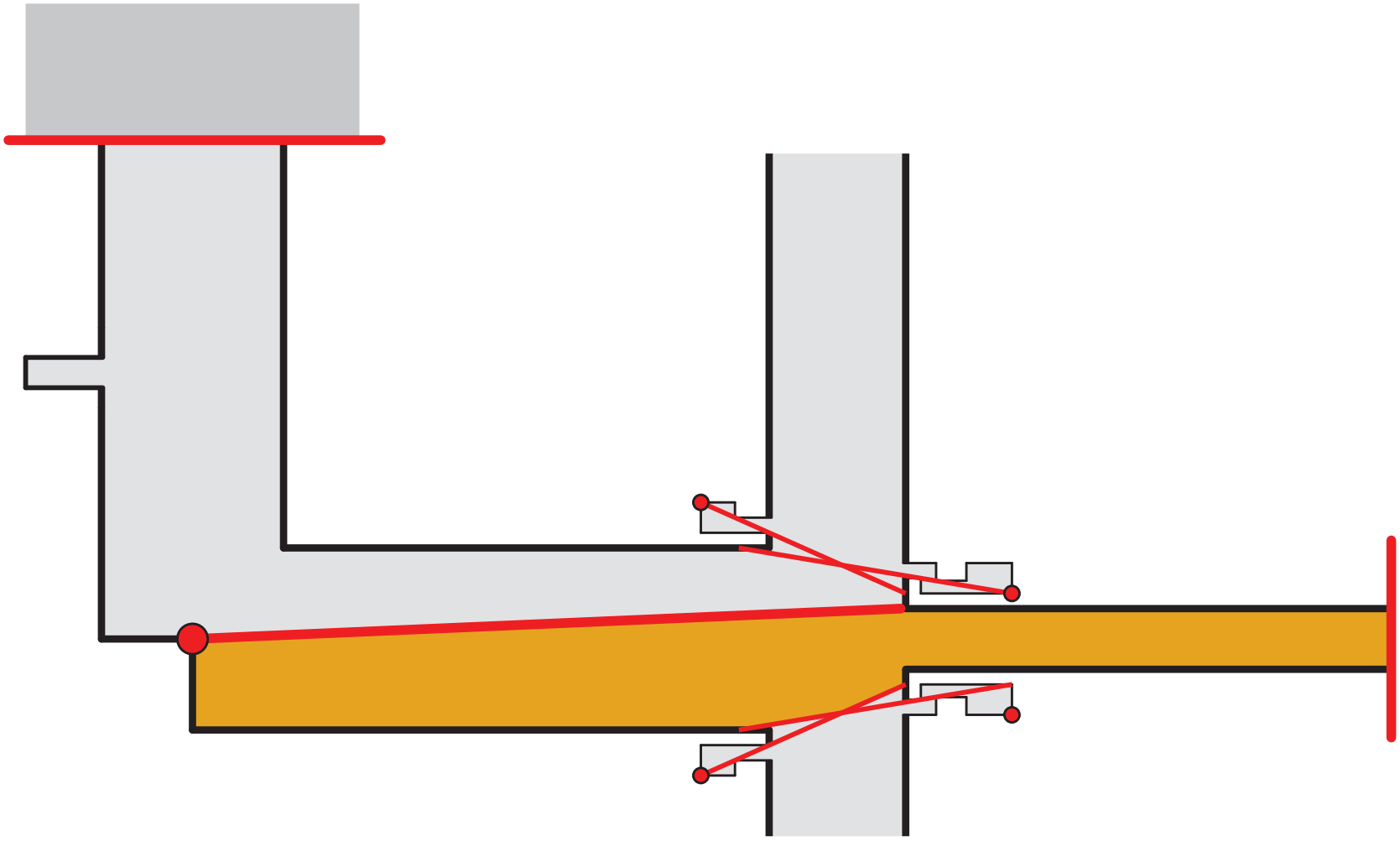
- Construction overview

Crossings between pipes and NCL edges



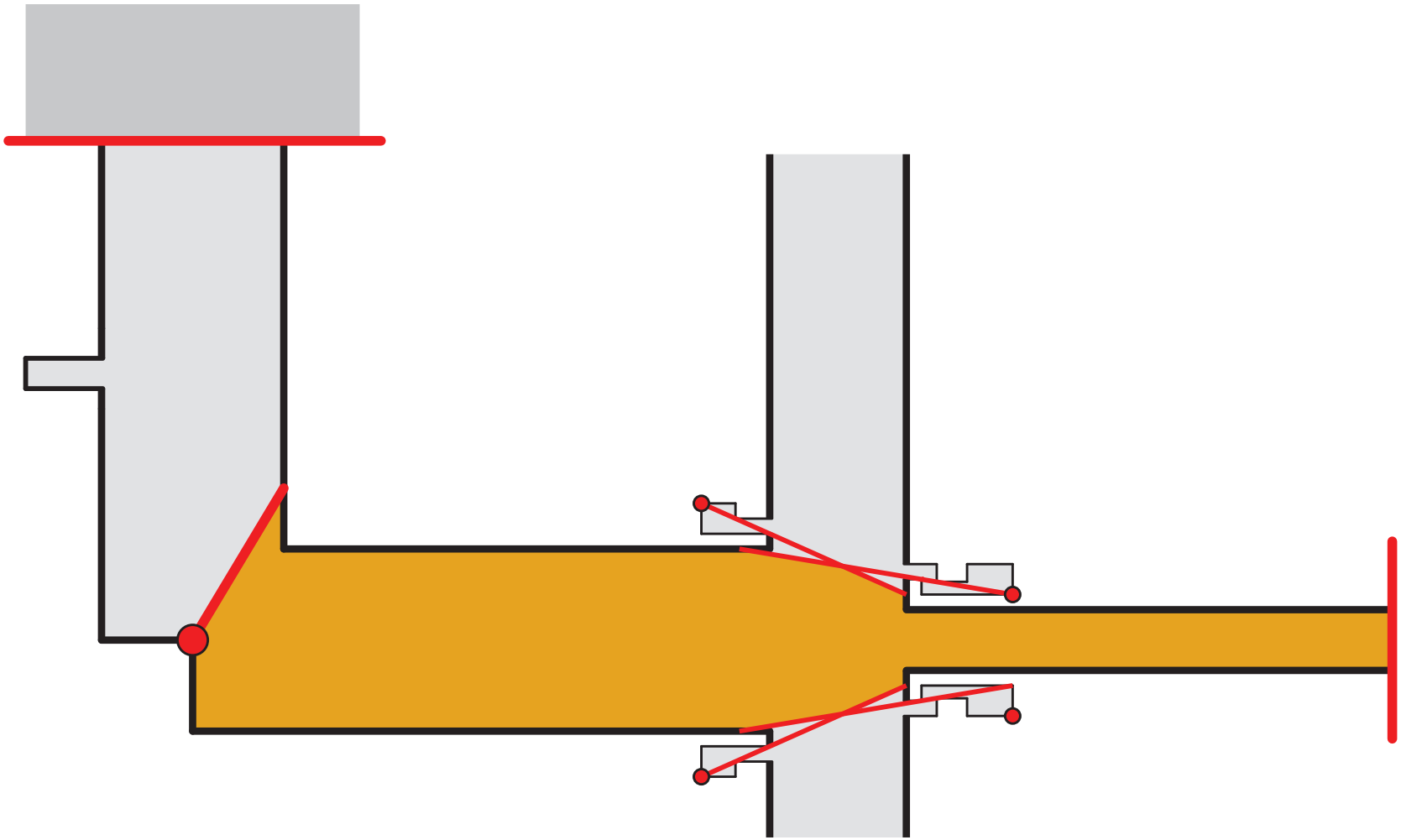
- External searchlights are closing the pipe's ends

Crossings between pipes and NCL edges



- Search begins

Crossings between pipes and NCL edges



- Search continues

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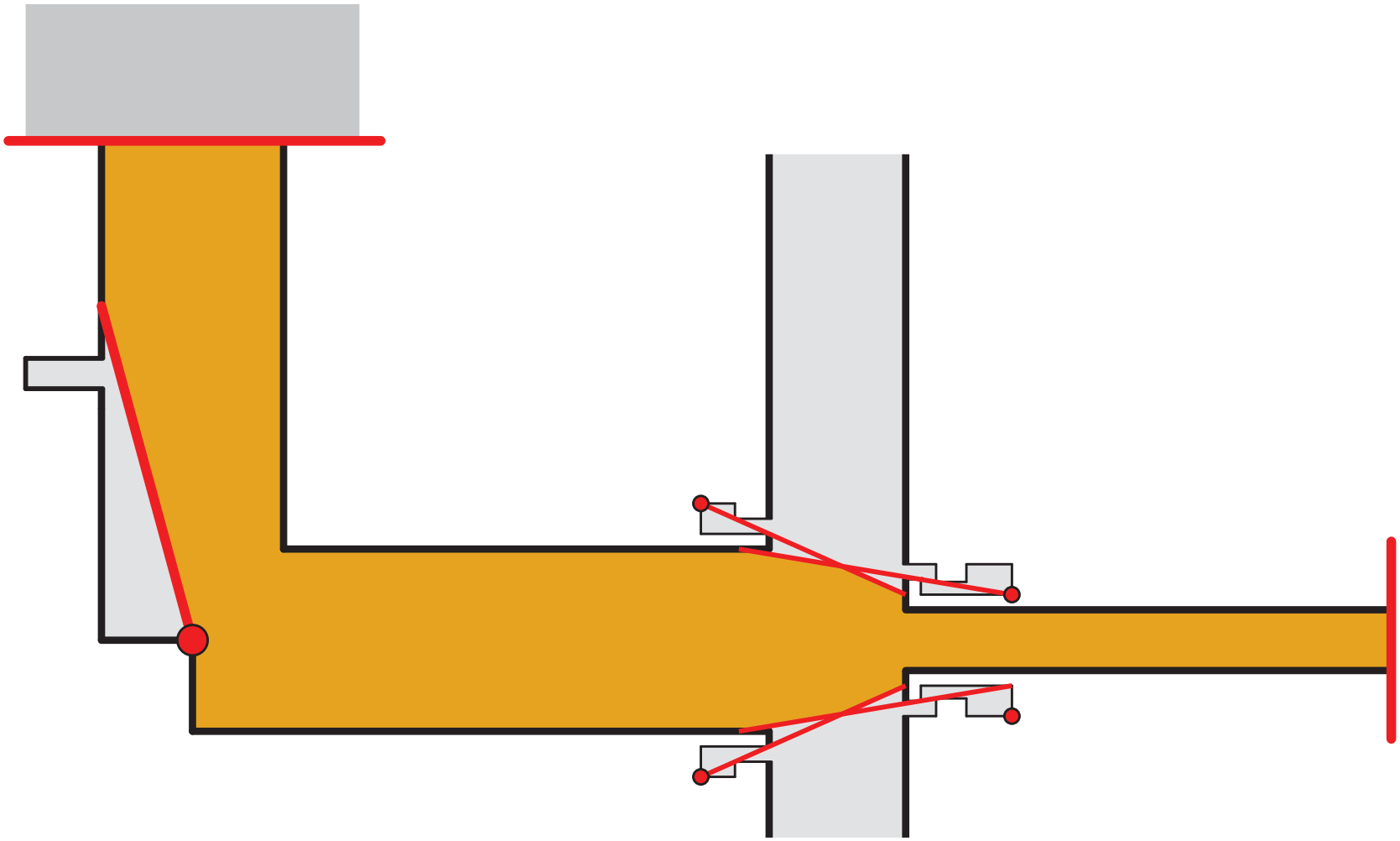
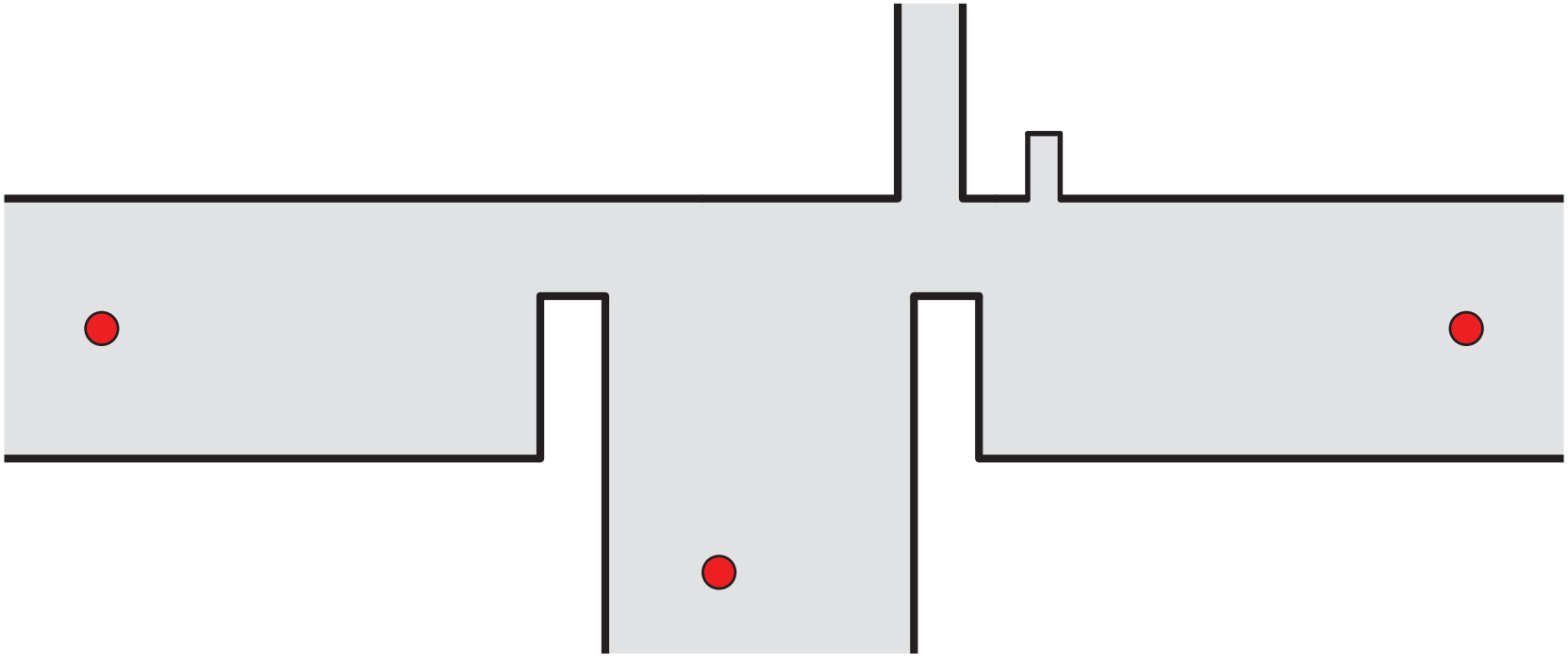


- Search ends

OR nodes



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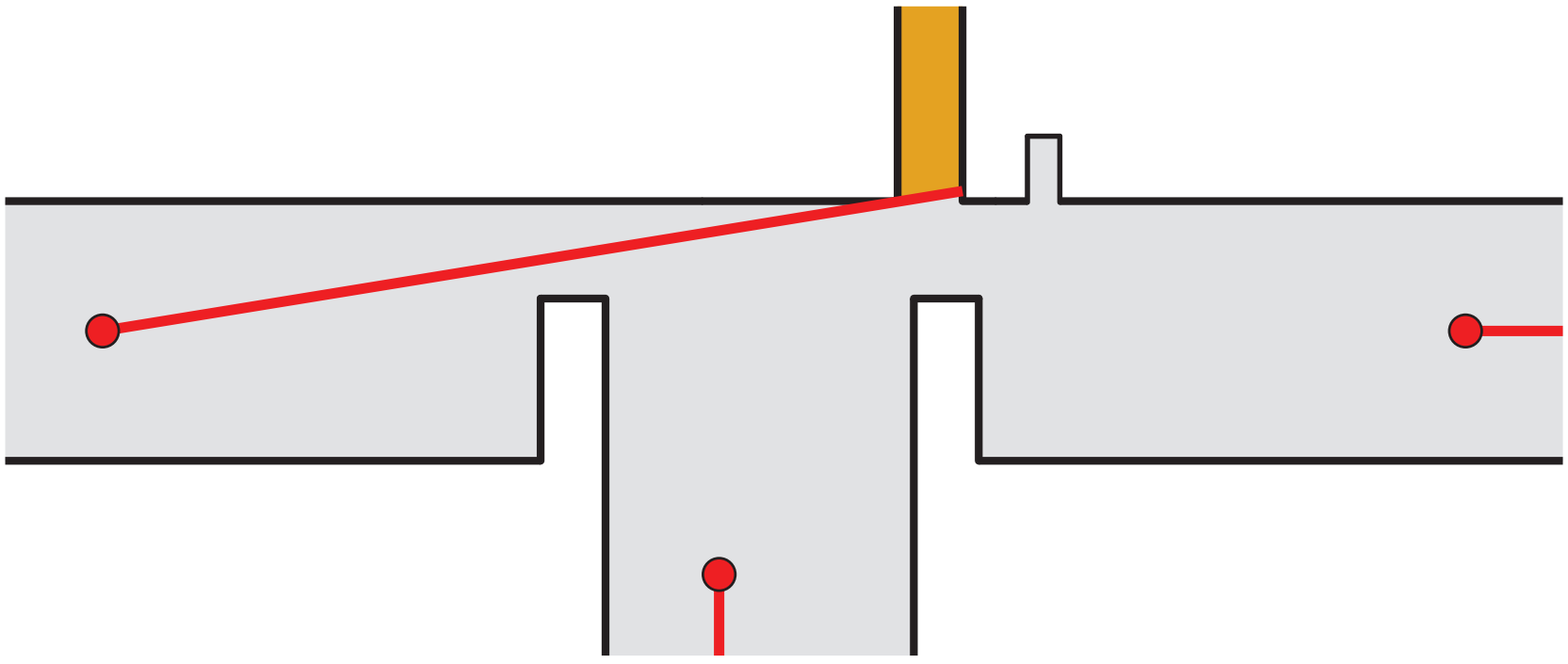

- Legal configuration (pipe still clear)

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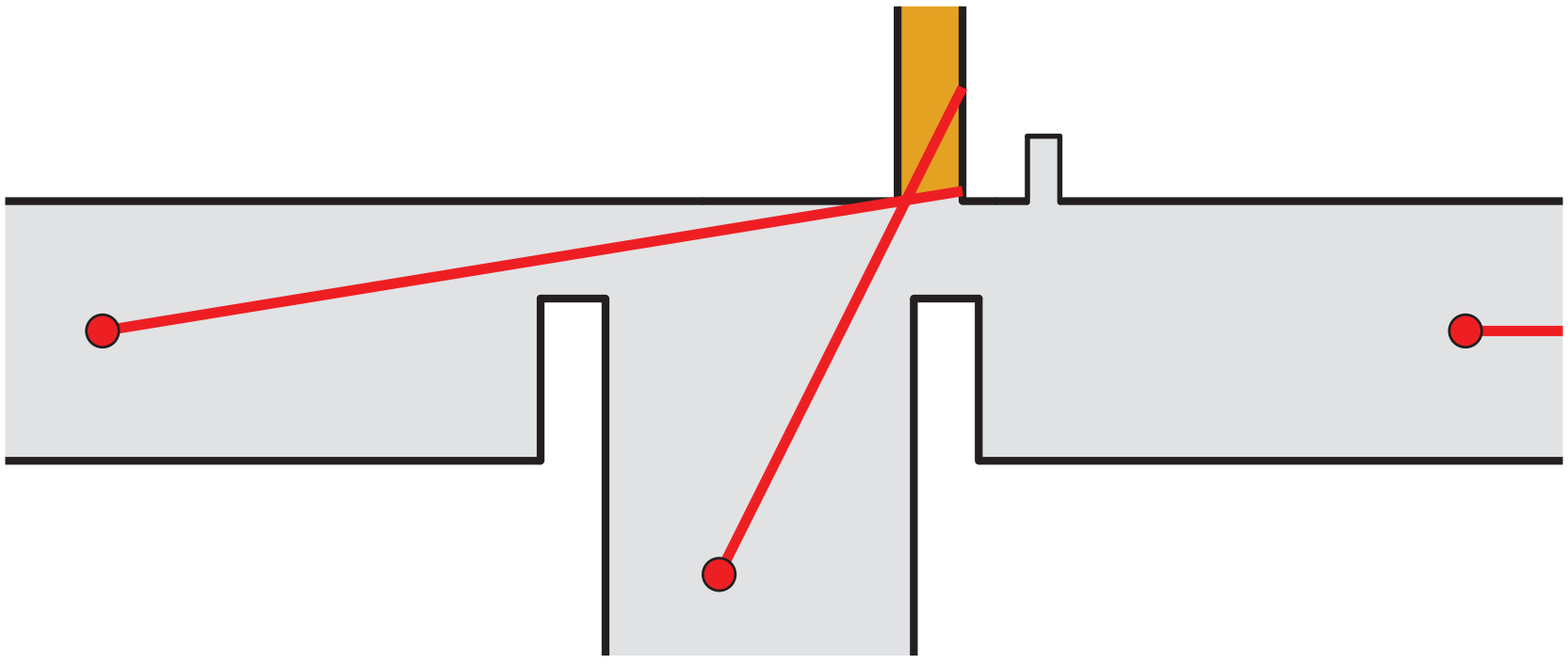

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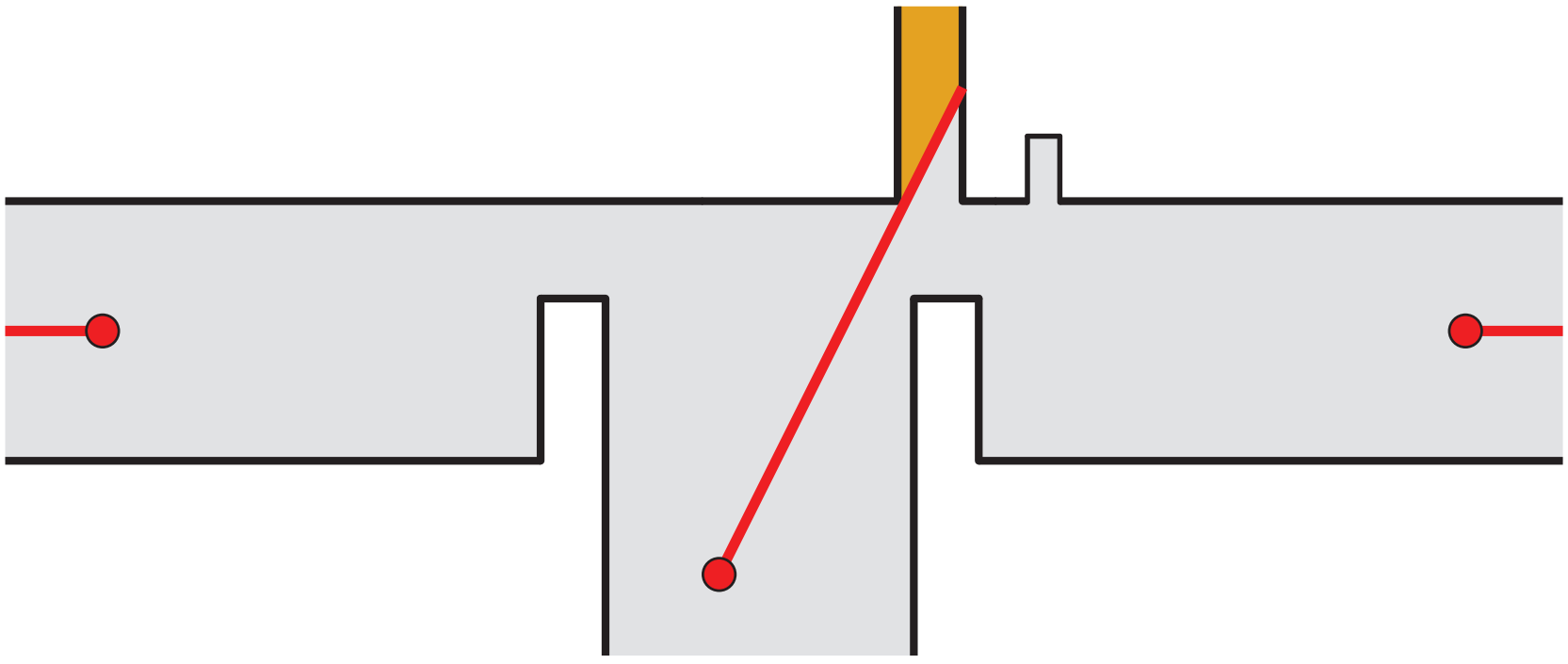

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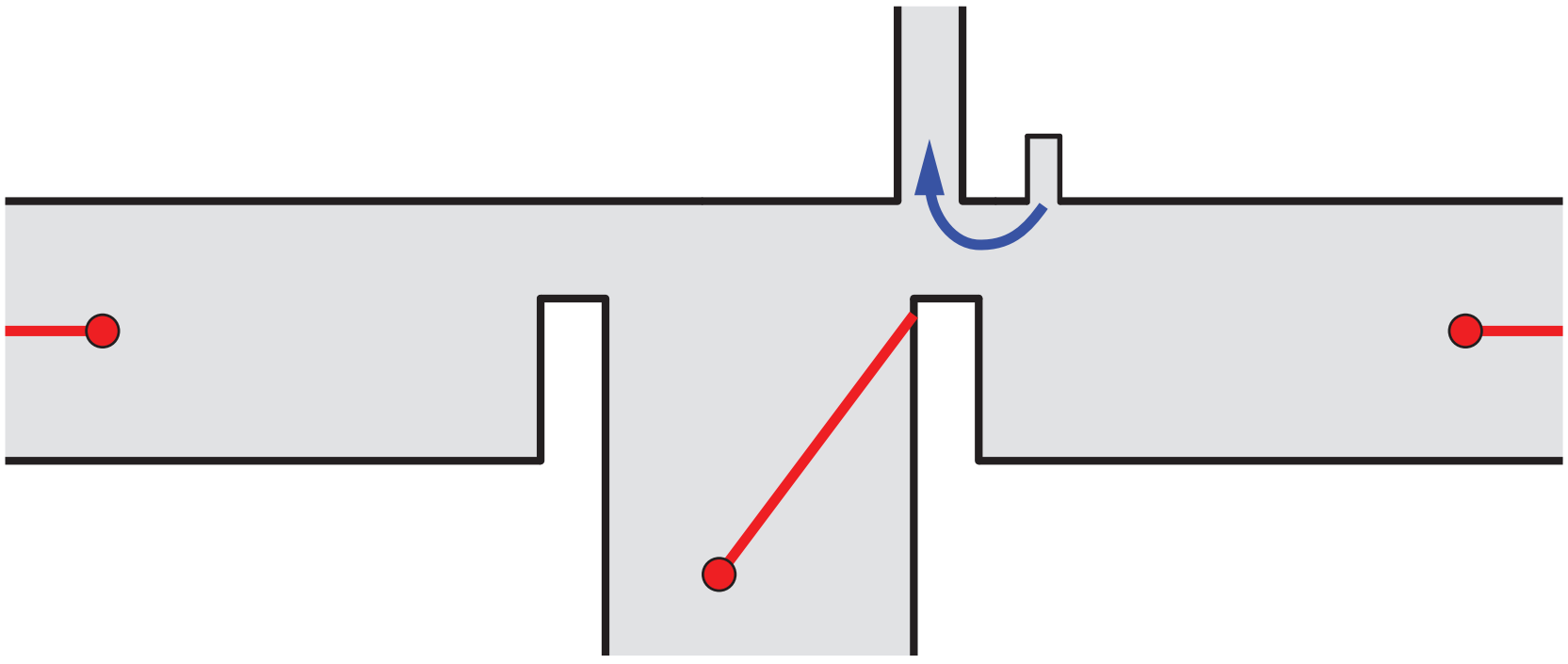
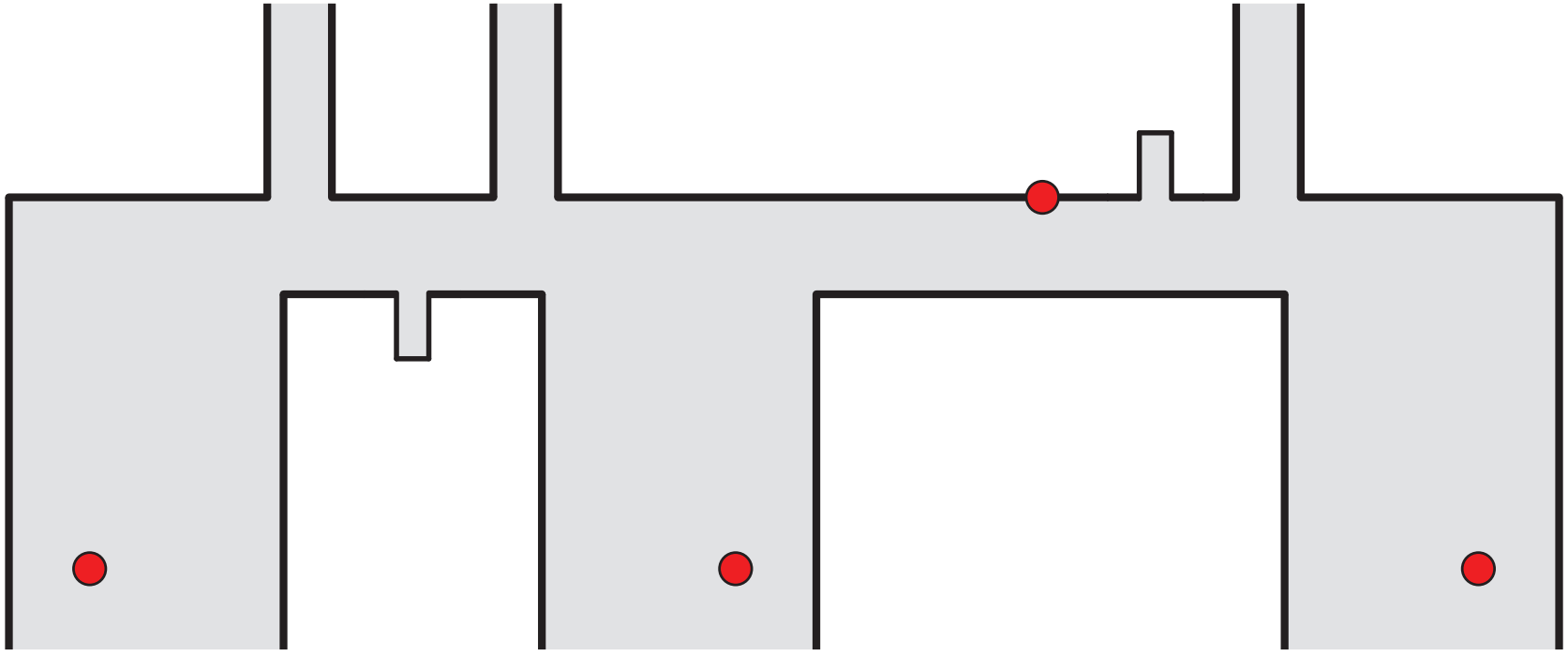


- Illegal configuration (pipe recontaminated)

AND nodes



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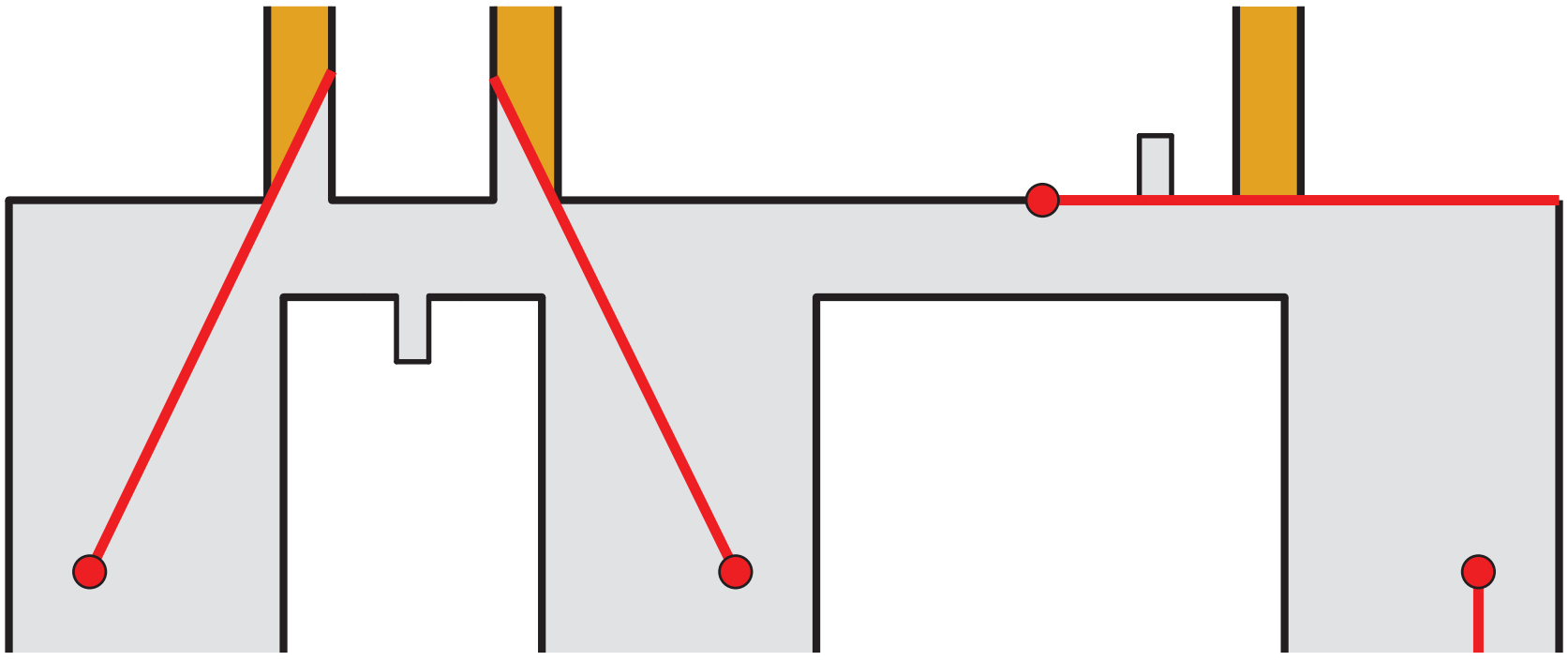

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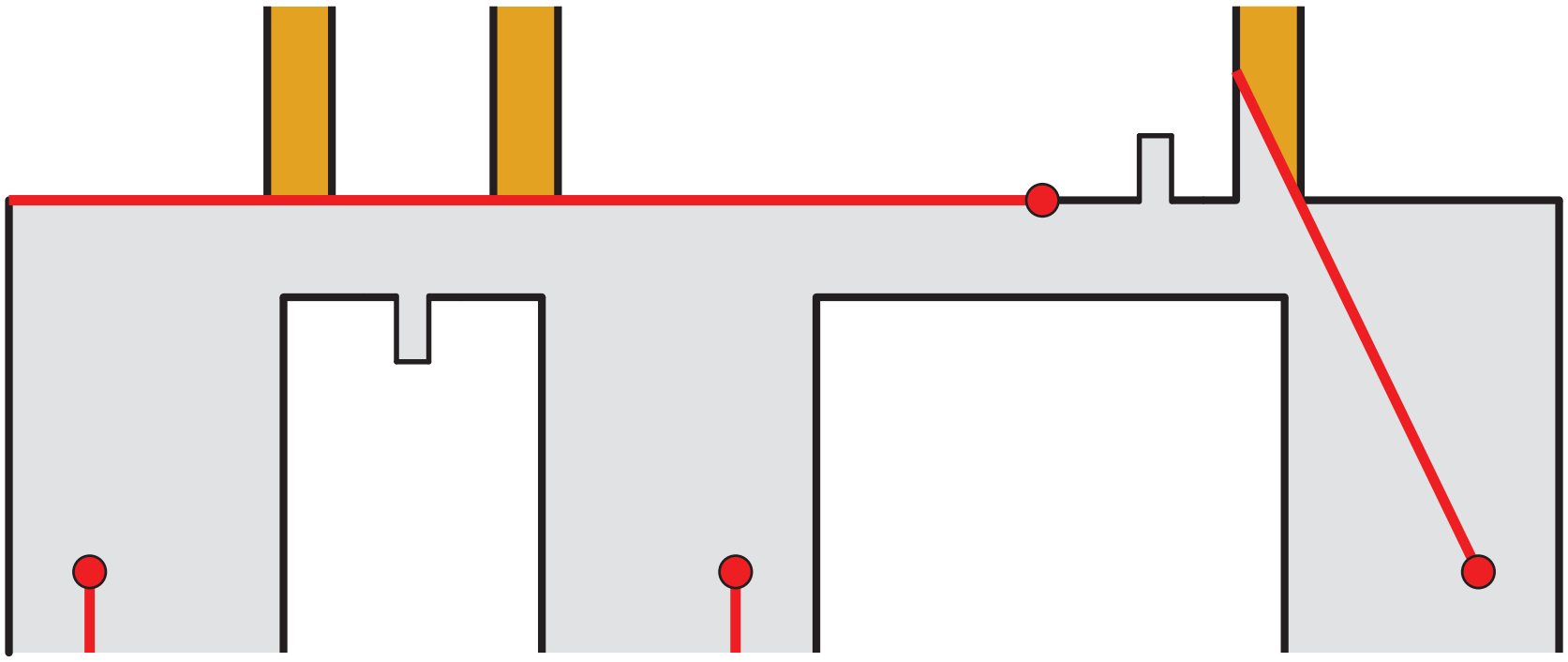

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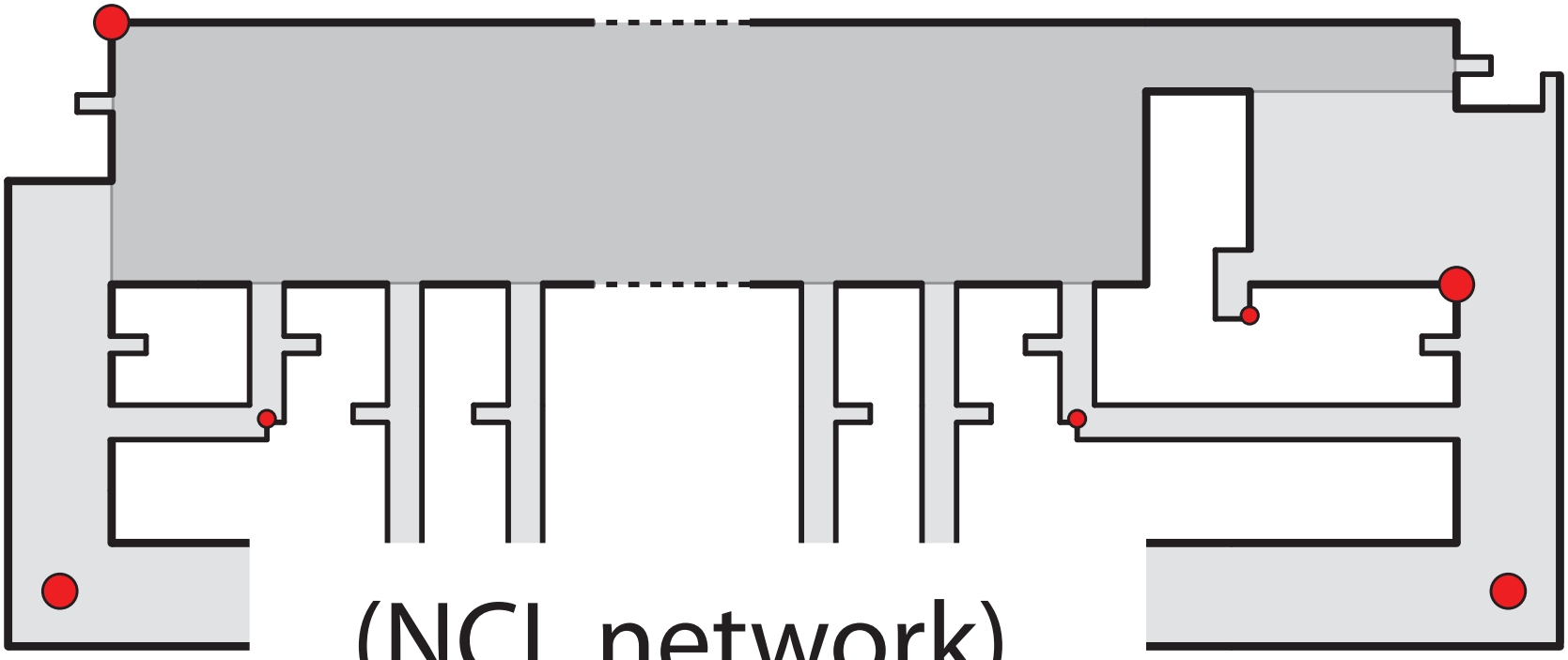
Clearing process



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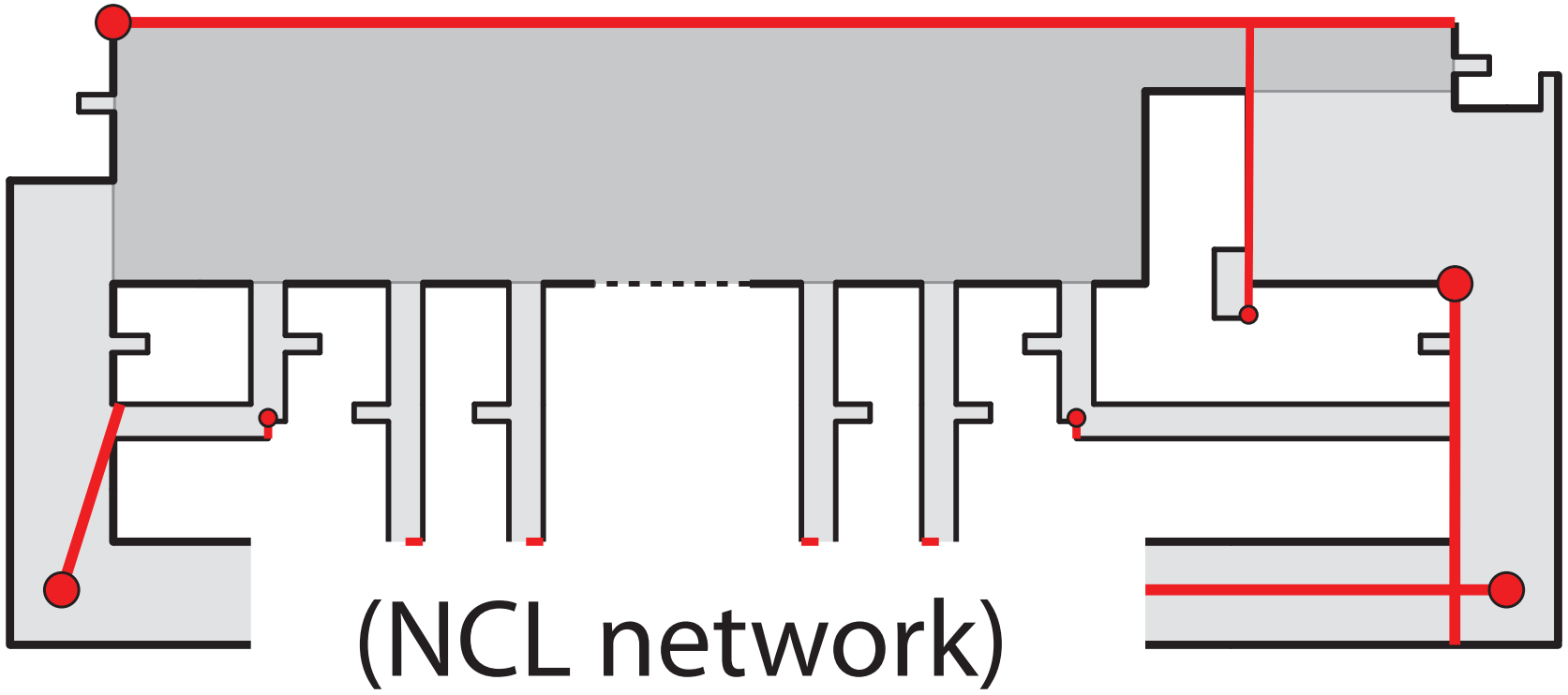

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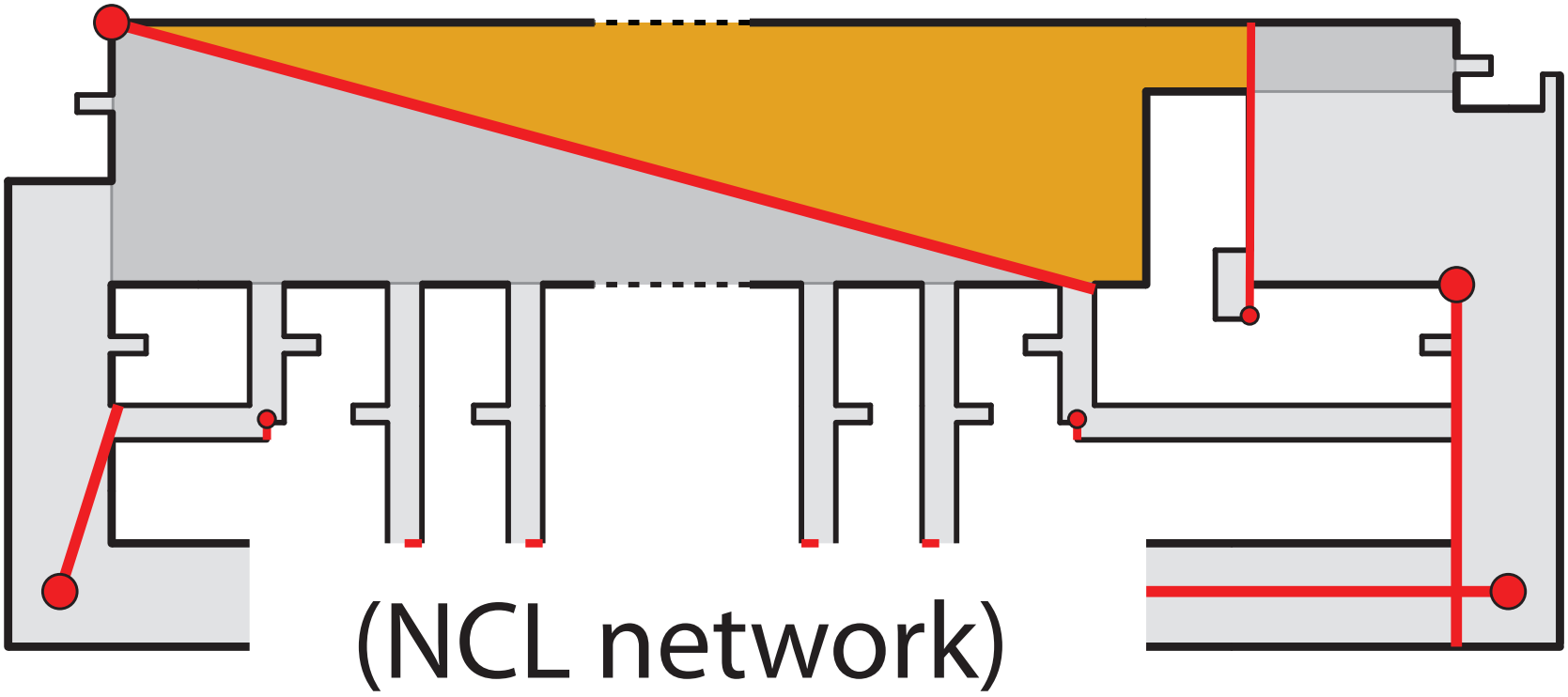

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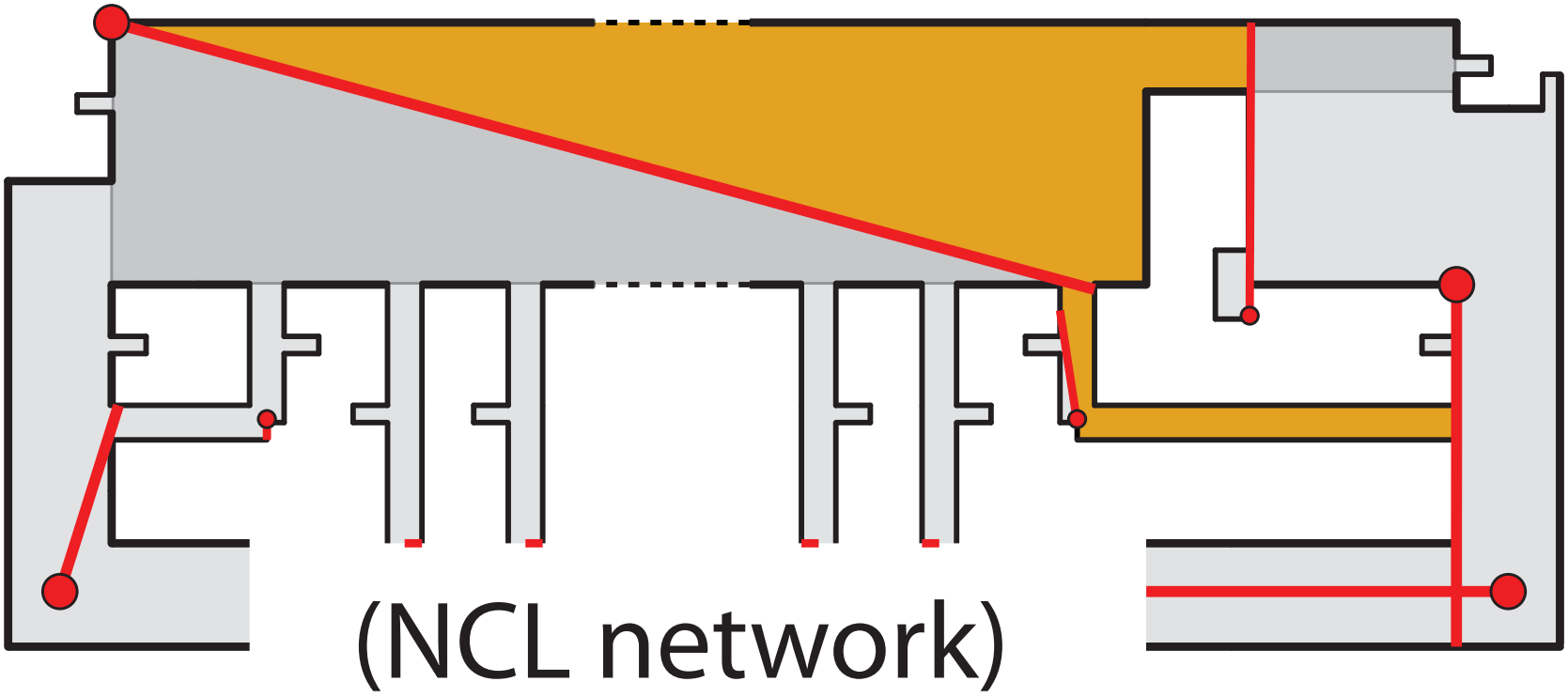

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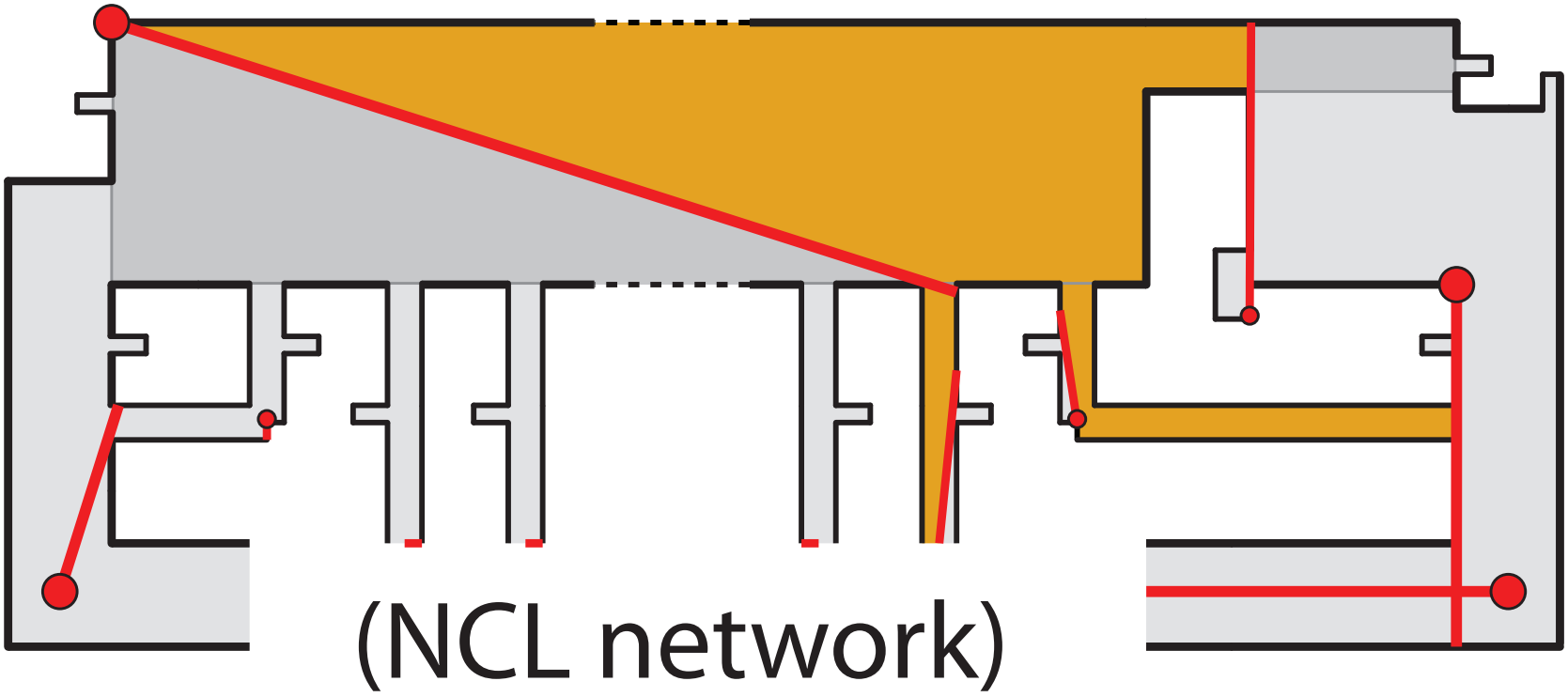

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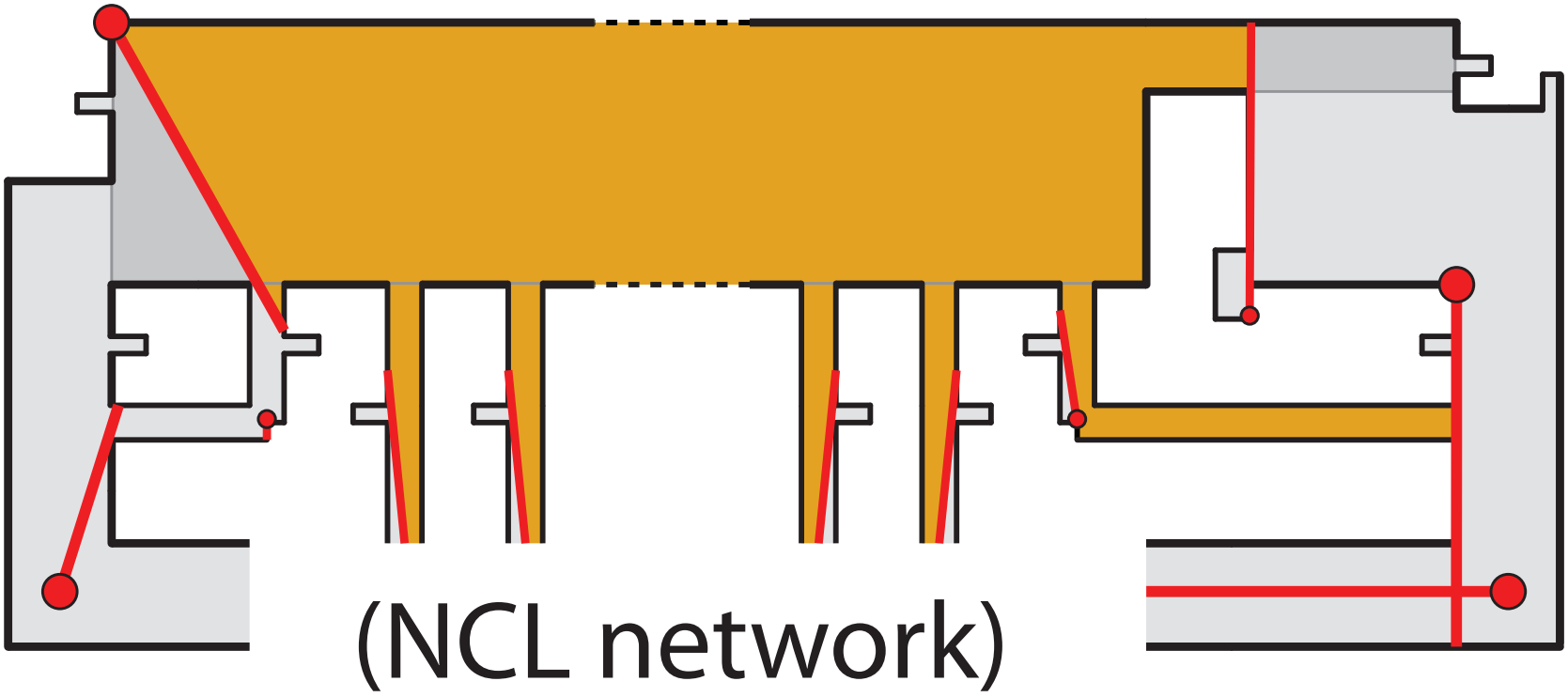

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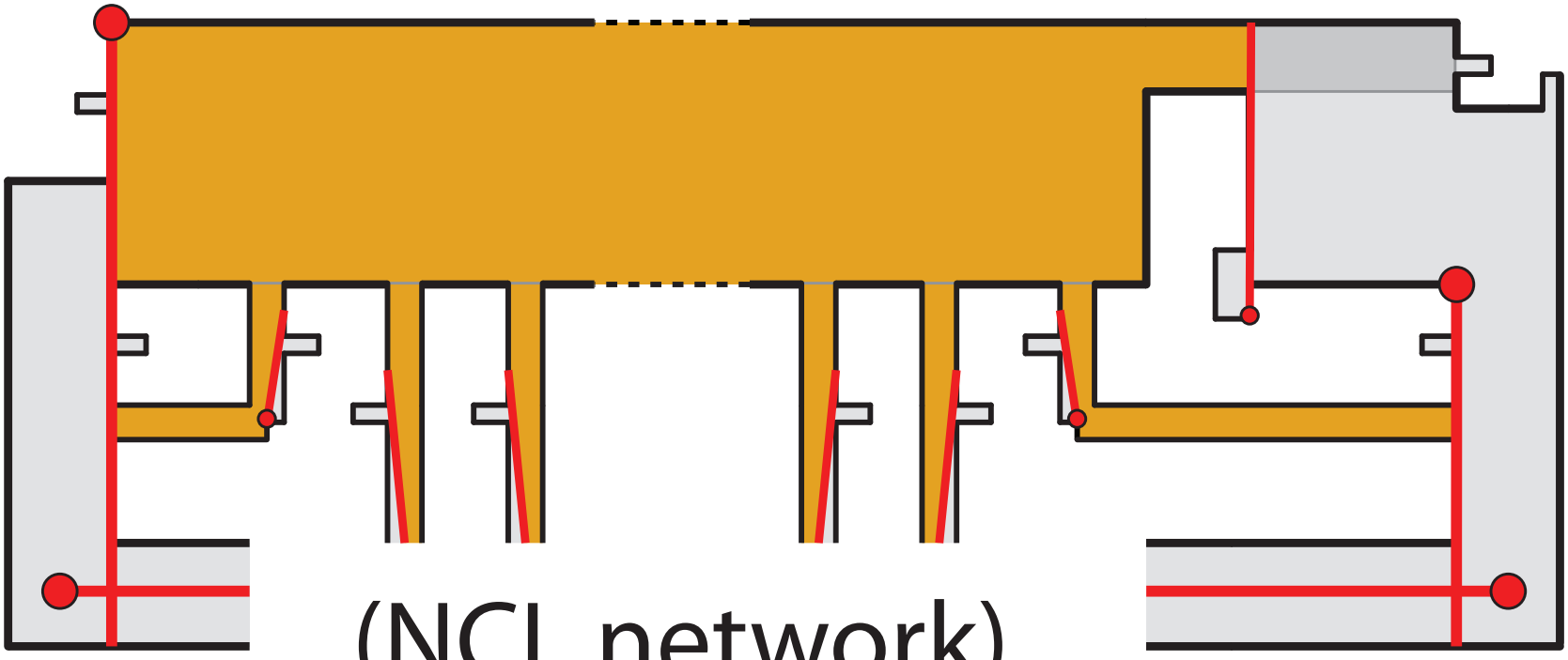

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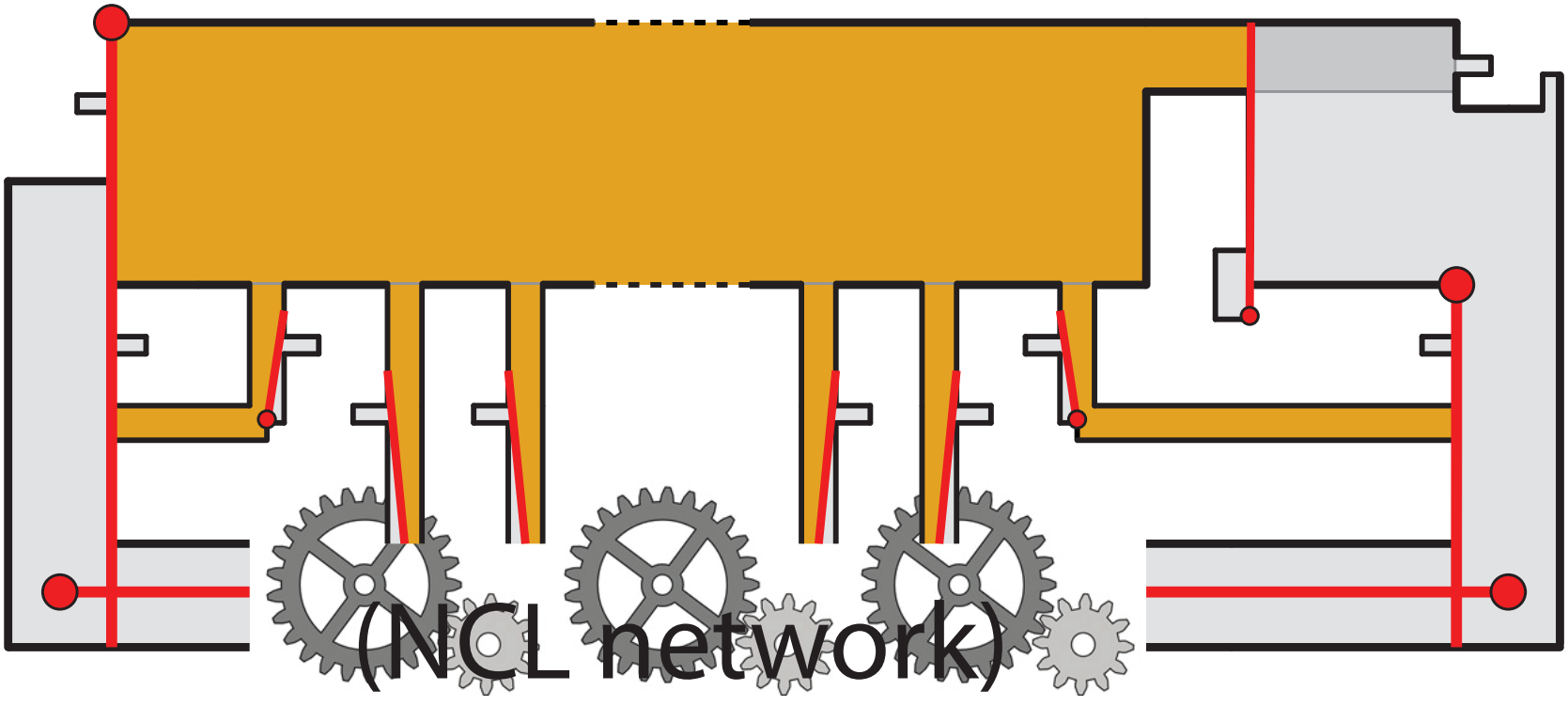

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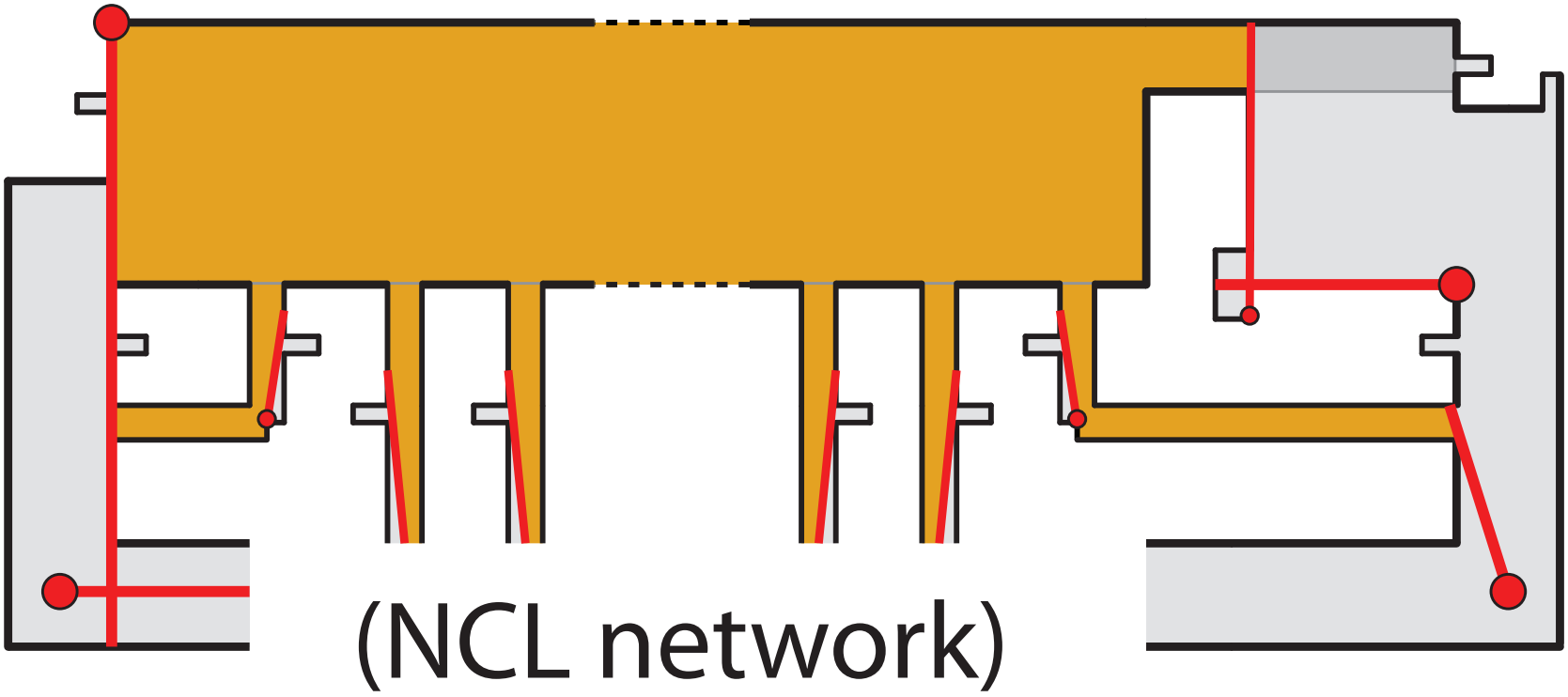

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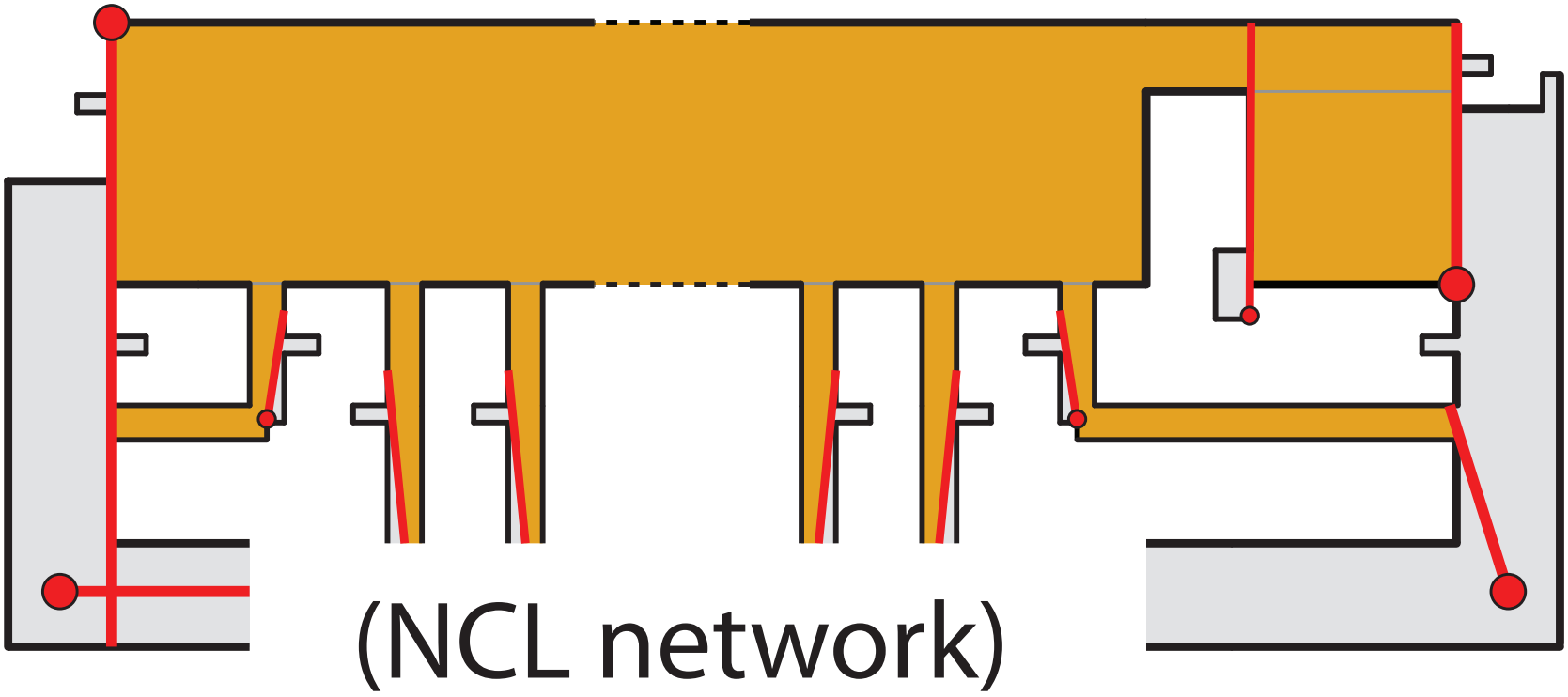

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  Is it also **PSPACE**-complete?
- Generalizing to 3D polyhedral environments, Partial Searchlight Scheduling stays (obviously) **PSPACE**-hard, but is it in **PSPACE**?
- 3D Full Searchlight Scheduling is **NP**-hard for general polyhedra, but is it in **NP**?
  What about orthogonal polyhedra?

References

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