Shape Formation by Programmable Particles

Brief Announcement – DISC 2017

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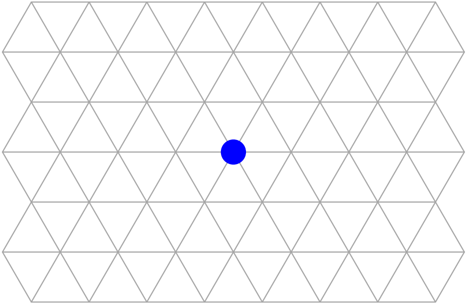
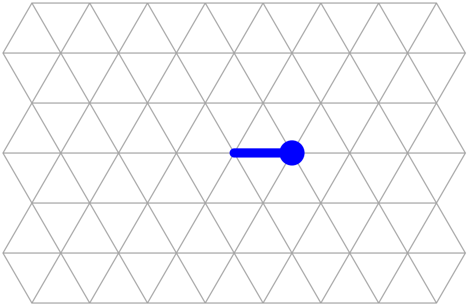
Amoebots

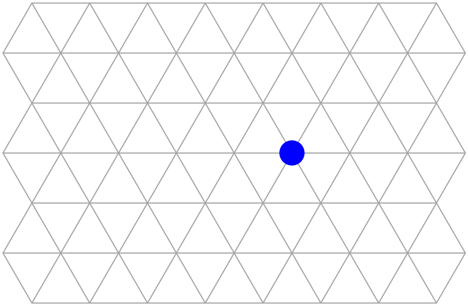


In this model, particles occupy nodes of a hexagonal grid.



A particle can move by *expanding* and *contracting*.

Amoebots



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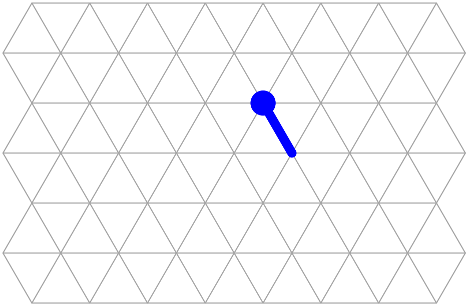

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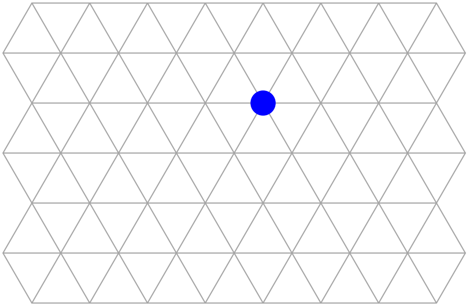

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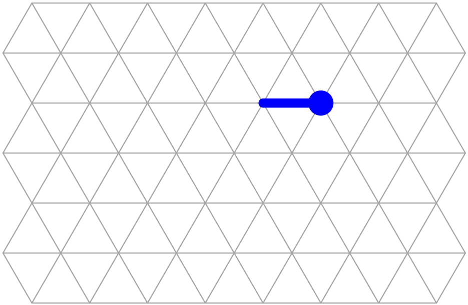

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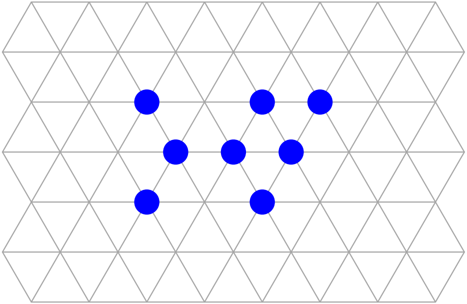
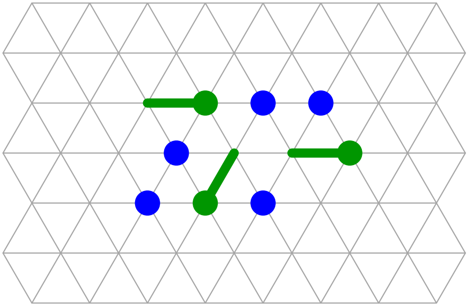
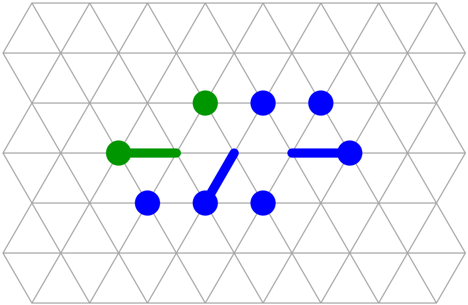


A *system* of particles is given.



Particles move *asynchronously*.

Amoebots



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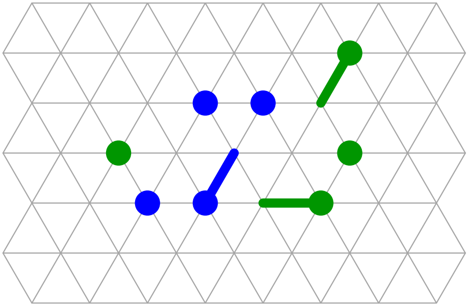
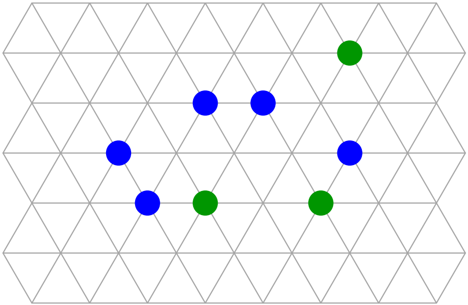


At each step, any set of particles is activated by an *adversary*.

Amoebots



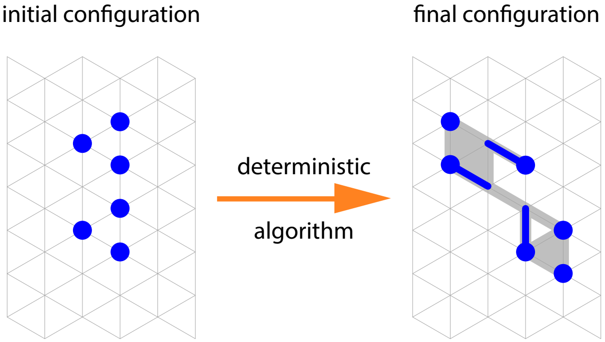
At each step, any set of particles is activated by an *adversary*.

Shape Formation

final shape

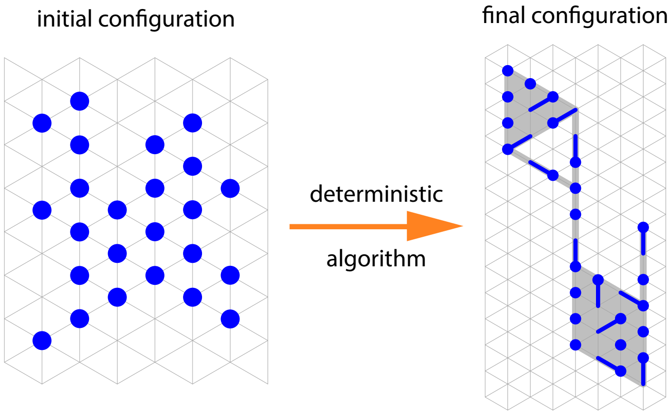
The goal is to form a *shape* that is given as input to all particles.

Shape Formation



The shape formation algorithm should be *deterministic*.

Shape Formation



The shape can be scaled up depending on the size of the system.

Related Literature

Original Amoebot paper:

Z. Derakhshandeh, R. Gmyr, T. Strothmann, R.A. Bazzi, A.W. Richa, and C. Scheideler
Leader election and shape formation with self-organizing programmable matter
In *Proceedings of 21st International Conference on DNA Computing and Molecular Programming (DNA)*, 117–132, 2015

Randomized shape-formation algorithm for sequentially activated Amoebots starting from a triangular shape:

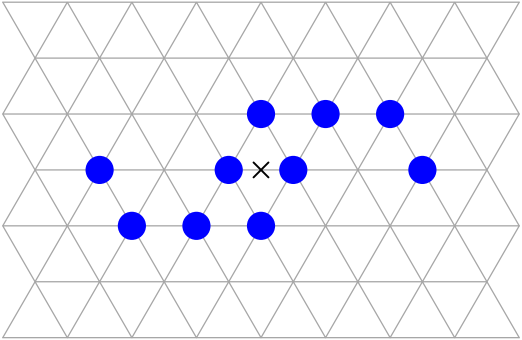
Z. Derakhshandeh, R. Gmyr, A.W. Richa, C. Scheideler, and T. Strothmann
Universal shape formation for programmable matter
In *Proceedings of the 28th ACM Symposium on Parallelism in Algorithms and Architectures (SPAA)*, 289–299, 2016

Our Particle Model

The n particles in the system:

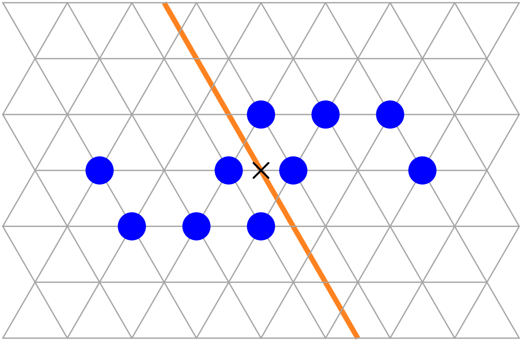
- initially form any simply connected shape
- know the final shape but do not know n
- have a constant amount of internal memory
- are anonymous and start in the same state
- can communicate with adjacent particles
- do not have a compass
- may not agree on a clockwise direction
- are activated asynchronously
- execute the same deterministic algorithm
- cannot occupy the same node

Unbreakable Symmetry



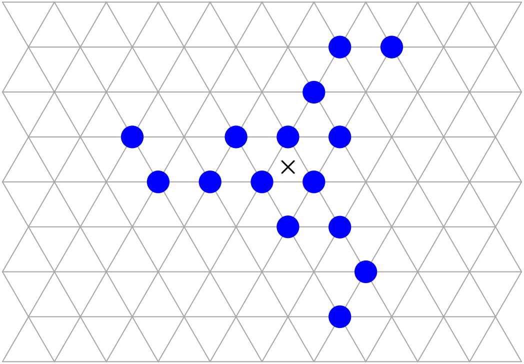
If the system has a center of symmetry not in a grid node...

Unbreakable Symmetry



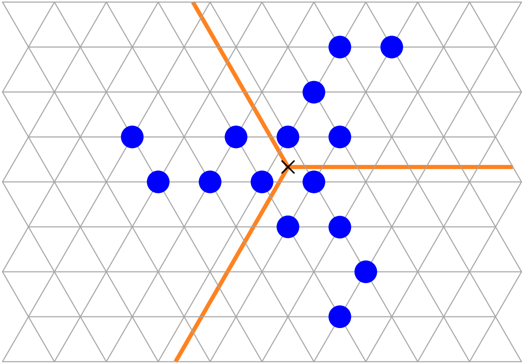
Then this symmetry is impossible to break.

Unbreakable Symmetry



The same holds for systems with a 3-fold rotational symmetry.

Unbreakable Symmetry



If the center is not in a grid node, they symmetry is unbreakable.

Statement of Results

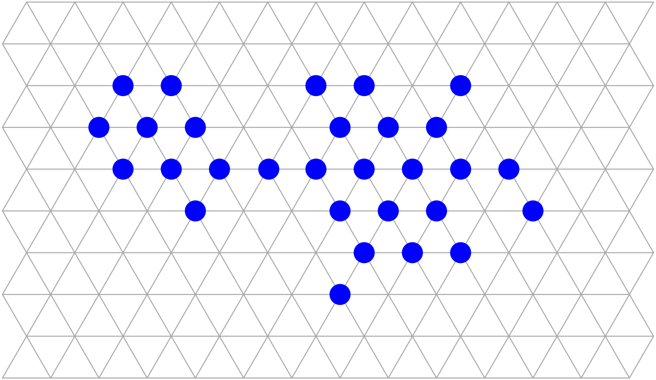
Theorem

If the system initially has an unbreakable symmetry, it cannot form shapes that do not have the same symmetry.

Theorem

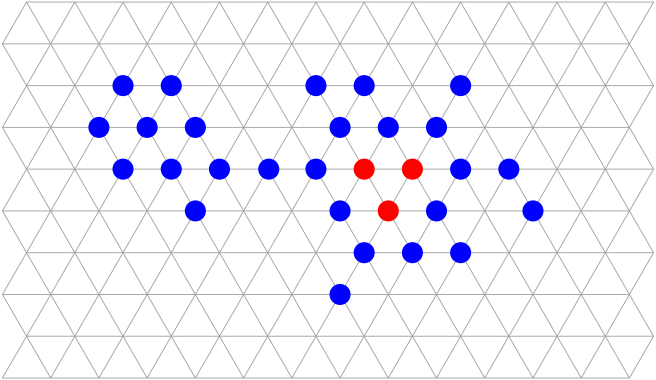
For all other cases, there is a universal shape-formation algorithm, provided that the system is sufficiently large. The particles perform $O(n^2)$ moves in total and terminate in $O(n^2)$ rounds.

Universal Shape-Formation Algorithm



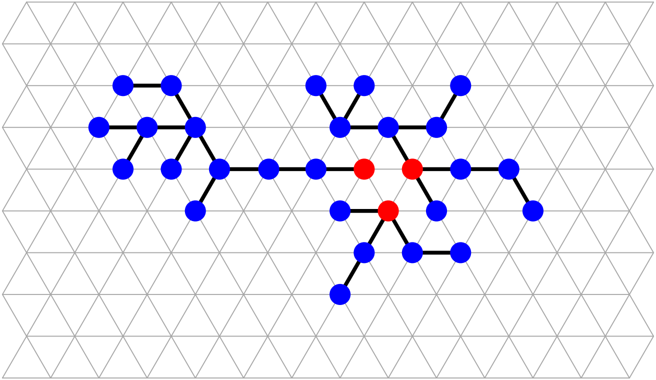
Start with a sufficiently large simply connected system.

Universal Shape-Formation Algorithm

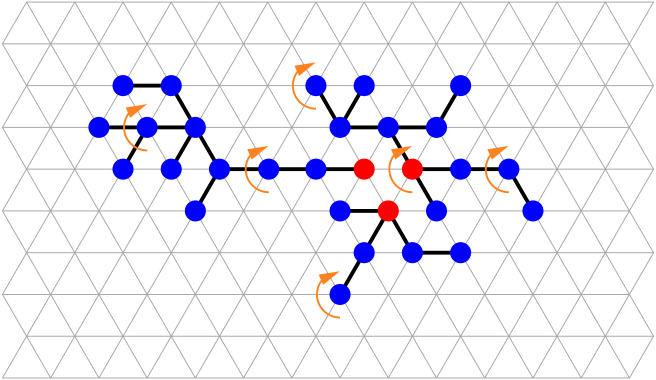


Phase 1: attempt to elect a leader.

Universal Shape-Formation Algorithm

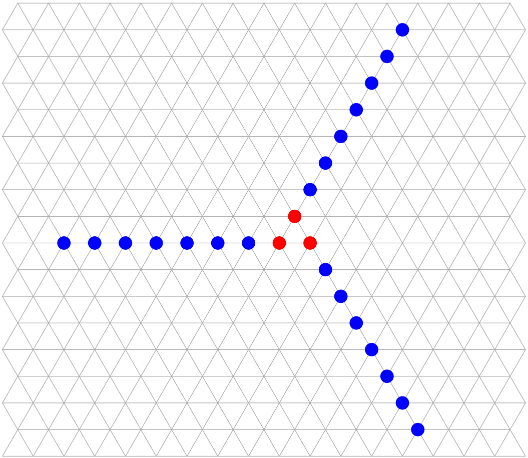


Phase 2: construct a spanning forest.



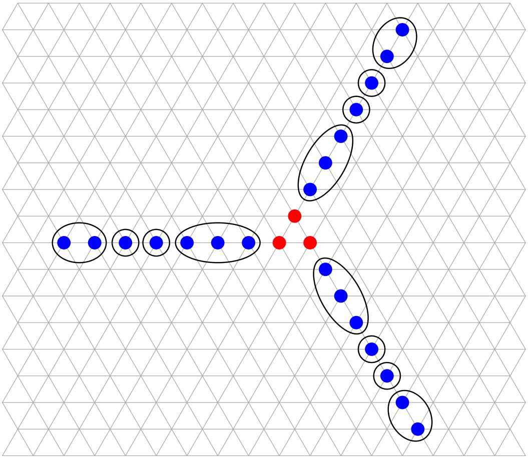
Phase 3: agree on a clockwise direction.

Universal Shape-Formation Algorithm



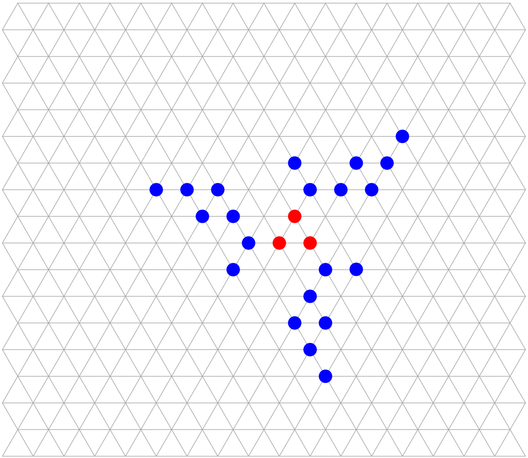
Phase 4: form one line per leader.

Universal Shape-Formation Algorithm

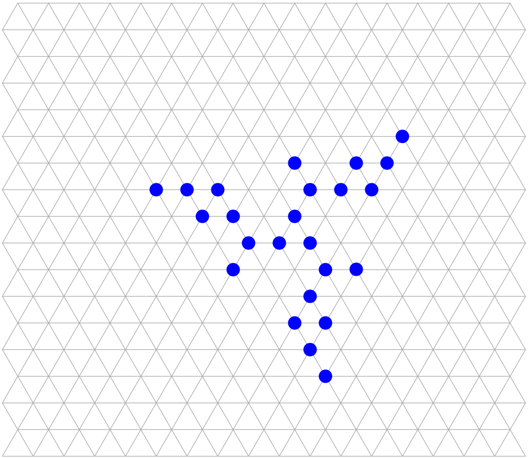


Phase 5: determine the scale of the final shape and assign roles.

Universal Shape-Formation Algorithm

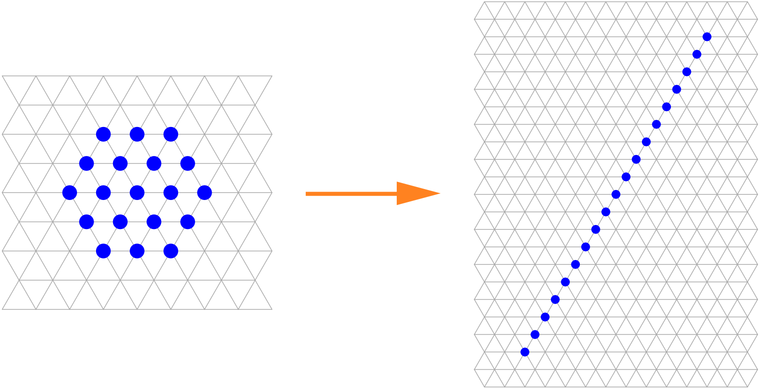


Phase 6: form the final shape.



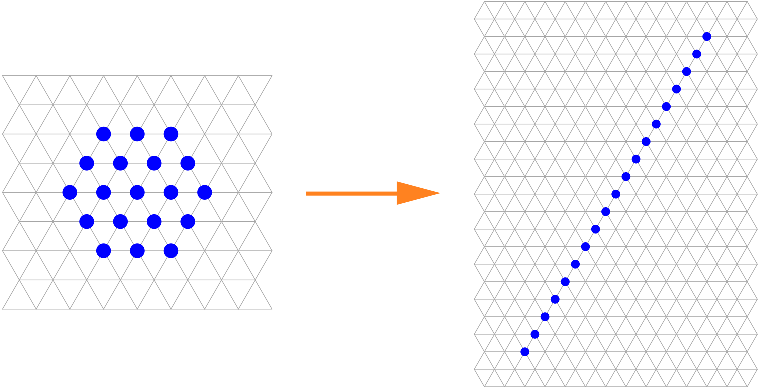
Phase 6: form the final shape.

Matching Lower Bound



This example shows that $O(n^2)$ total moves are optimal.

Matching Lower Bound



Open problem: are $O(n^2)$ rounds optimal?