

# The 3-dimensional Searchlight Scheduling Problem

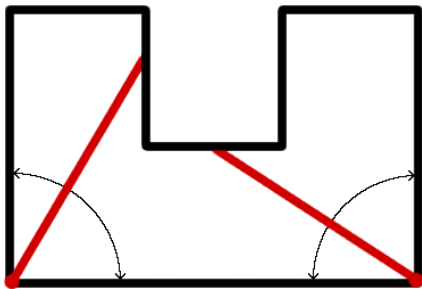
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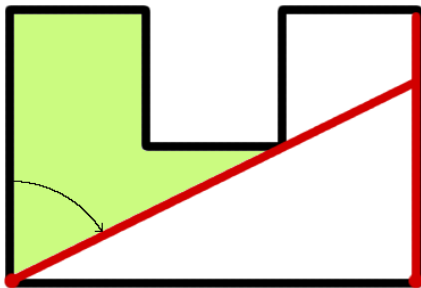
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**Suzuki & Yamashita, 1990:** A polygon has to be searched by stationary guards, each carrying an orientable laser beam. The intruder can move arbitrarily fast. Does a searching schedule exist that always catches the intruder?



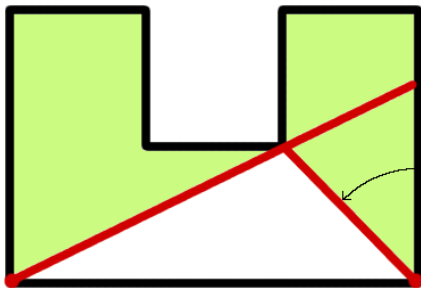
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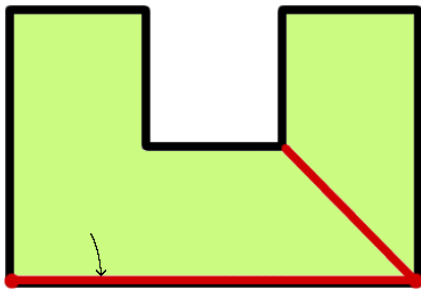
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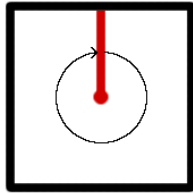
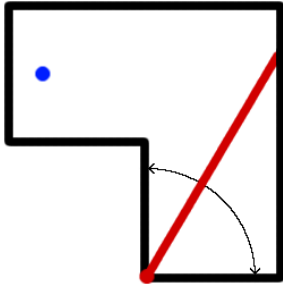
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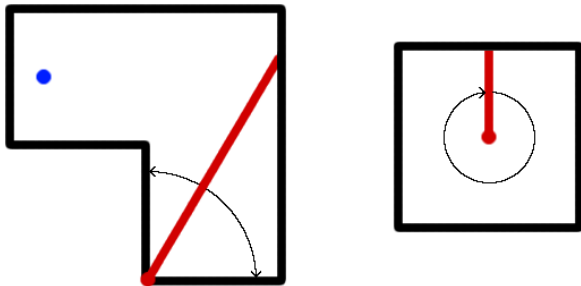
# Solvability of SSP instances

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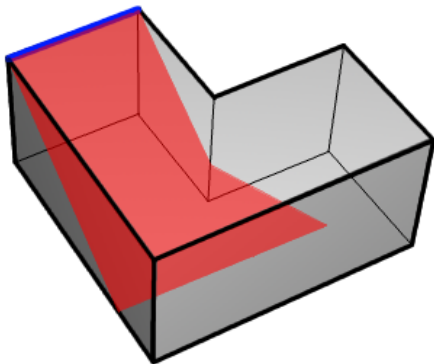
If all guards lie on the border and every point in the polygon can be seen by at least one guard, then the instance is solvable.

**SSP:** Is a given instance (polygon + guards) solvable?

Determining if SSP is NP-hard is a long-standing open problem.

## 3D searchlights

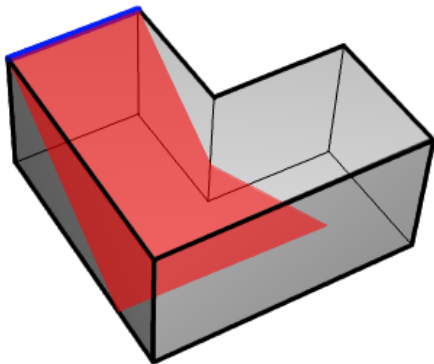
**3SSP:** Guards are line segments contained in a polyhedron, and searchlights are half-planes emanating from such segments.





## 3D searchlights

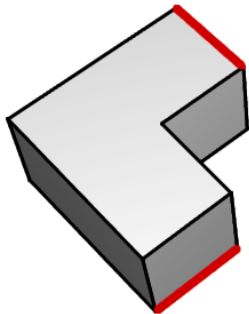
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- Line-guards in 3D act like point-guards in 2D.
- Searchlights still have 1 degree of freedom.
- $\text{SSP} \leq_{\mathbf{P}} \text{3SSP}$  trivially.

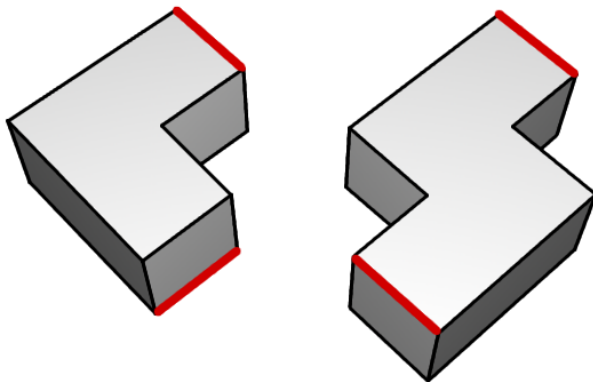
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However, instances with only one guard are solvable if and only if the guard lies on the boundary and can see the whole interior. (...)

**T3SSP:** Decide if a 3SSP instance is solvable within a given time, where searchlights have bounded angular speed.

Claim:  $3SAT \leq_P T3SSP$ .

# Timed 3SSP is NP-hard

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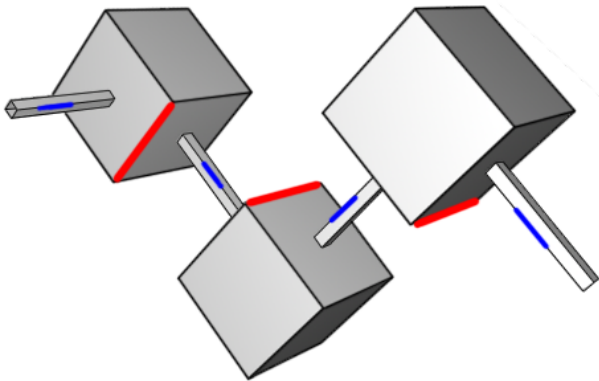
We will convert a 3CNF formula  $F$  into an instance of 3SSP which is solvable in 3 seconds if and only if  $F$  is satisfiable, where the angular speed of guards is  $90^\circ/\text{sec}$ .

Moreover:

- the polyhedron may be chosen to be orthogonal,
- all the guards may be chosen to lie on edges.

# Chain of sticks and boxes

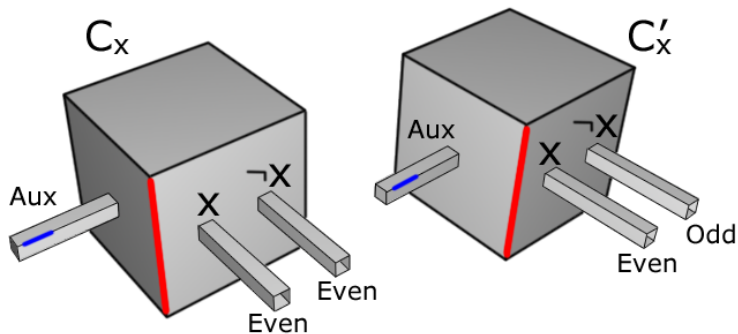
A stick can be cleared in 1 second, iff both its ends stay capped.



Sticks in the chain must be cleared in seconds 1 and 3, alternately.

# Boolean variable

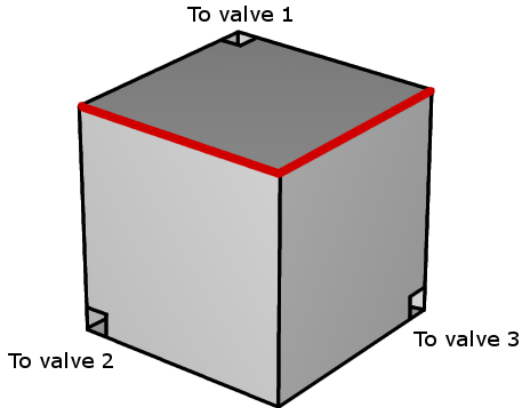
One pair of links for each occurrence of  $x$  in  $F$ .



The variable is true iff both red guards sweep in the same direction.

## 3-input OR gate

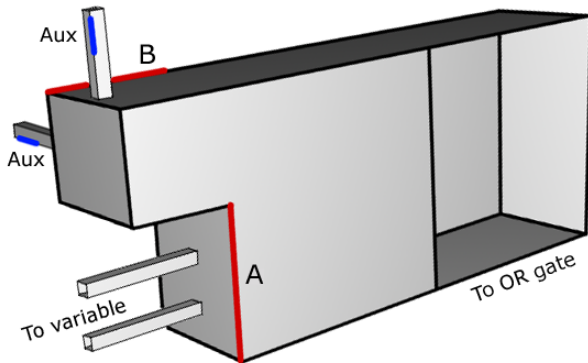
The cube can be cleared iff at least one of the valves is closed.





# Valve

If the variable occurrence is satisfied, both links can be cleared simultaneously, and guard *A* can turn to close the valve. Otherwise, guard *A* is forced to cap the links, and can't close the valve.



In both cases, guard *B* clears the valve during second 2.

## Further work

- Characterize instances of 3SSP solvable by 2 guards.
- Minimize motion instead of time.
- Study different visibility models.
- Is T3SSP approximable?
- Is T3SSP PSPACE-hard?
- Is 3SSP (without time constraints) NP-hard?
- Is SSP NP-hard?

**Thank you!**



J. O'Rourke.

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